From Test Collisions to Stiffness Coefficients

Jon Neades. AiTS, South Cerney, Glos.

Abstract
The use of computer programs to estimate the changes in velocity (Delta-V) suffered by a vehicle in a collision by estimating the amount of crush energy absorbed have been in use since the late 1970's These programs require a database of stiffness coefficients which define force per unit of crush on the damaged vehicle. This paper explains the basic method used to establish such coefficients from test collisions performed from known speeds.

Background
The use of personal computers running a CRASH3 derivative program, such as EDCRASH or Ai Damage is now commonplace within the UK collision investigation fraternity. Documentation is available which describes the algorithms used and how they relate to practical situations.

The basic concept behind the CRASH3 programs is to compare the effect of test collisions with the crush produced in a real collision to determine the change in velocity which each vehicle suffers as a result. Obviously it is impractical to crash all known vehicles at all possible speeds and impact configurations to build up a simple comparative database of photographs. Instead the approach that has been adopted is to crash a number of vehicles at known speeds and use maths to determine other speeds and impact configurations. The basic equation used to determine the change in velocity, or Delta-V (Δv) for a vehicle as a result of the energy absorbed in crushing both the vehicles is given by the equation,

$$\Delta v_1 = \sqrt{\frac{2(E_1 + E_2)}{m_1(1+\frac{m_1}{m_2})}} \qquad \textbf{(A)}$$

m_1 and m_2 represent the masses of each vehicle
E_1 and E_2 represent the crush energy

A similar equation exists for vehicle two. Strictly speaking this equation only relates to collinear impacts where the force acting between the vehicles acts along the line joining the two centres of mass. In production programs, the equations used also contain terms which allow for non-collinear or non-central collisions as they are often termed. Non-central impacts produce rotation as well as a change in velocity.

Of interest is the reduction of equation (A) when one of the vehicles involved is very massive and absorbs no energy itself. Such a situation often exists in practice where a vehicle collides with a barrier. If m_2 is set to a very large number and E_2 set to zero equation (A) reduces to,

$$\Delta v_1 = \sqrt{\frac{2E_1}{m_1}} \qquad \textbf{(B)}$$

This is no more than a transposition of the well known kinetic energy equation ($E = \frac{1}{2} mv^2$). In these circumstances Δv_1 is known as the equivalent barrier speed (EBS) In other words the initial speed at which all the kinetic energy is converted into crush energy. Although of more historical interest, this demonstrates that if the energy of collision can be estimated for an immovable barrier impact, then the initial speed of the vehicle involved can be determined.

In the CRASH3 derivative programs rather more can be achieved than just calculating the EBS for vehicles. Equation (A) shows that the change in velocity of a vehicle can be calculated, regardless of its actual speed provided that an estimate of the energy absorbed can be made. This is a vital point, as a perceived shortcoming of equation (A), is that only the change in velocity can be calculated. The actual speed of the vehicle is indeterminable given just this information.

Before any of these equations can be used, a reliable method of determining the amount of energy absorbed in crushing the vehicle must be established. A value for the energy absorbed in the collision by each vehicle can be calculated from the expression,

$$E = L(AC + \frac{BC^2}{2} + G)(1 + \tan^2\theta) \qquad \textbf{(C)}$$

A and *B* are coefficients specified in the program
G = $A^2 / 2B$
C represents the crush depth
L = width of crush
θ = force angle from perpendicular

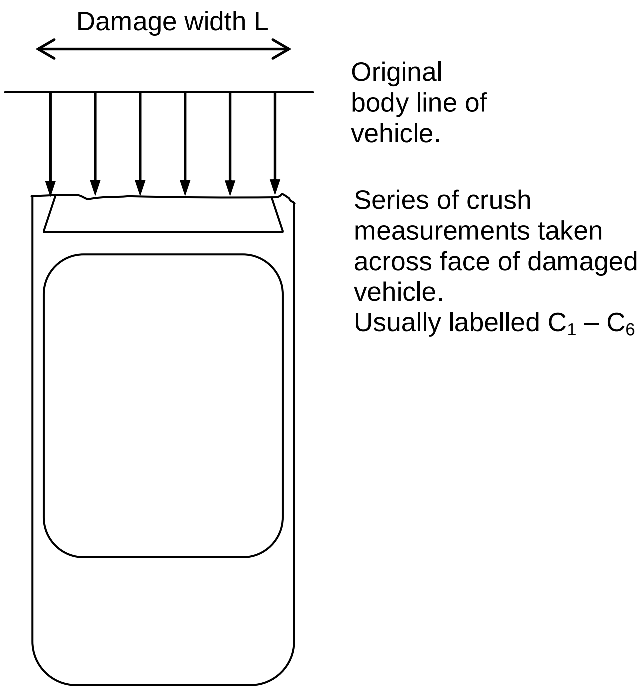
The derivation of all these equations, including the more complete versions is explained elsewhere[1]. What this paper seeks to address is the basic process by which the *A* and *B* coefficients are determined and why those particular coefficients are regarded more useful than perhaps a more obvious solution.

Crash testing

In its most primitive form, crash testing consists of propelling a vehicle head-on at a known speed into a solid barrier and measuring the residual crush as shown in Figure One. In practice there are a wide variety of impact configurations which are used.

Speeds at impact tend to be in the range 25 to 40 mph (40 – 65 kmh^{-1}) because statistically this is the speed band at which most injury accidents occur. In some tests, the impact is into another vehicle or perhaps a deformable barrier, such as with the NCAP testing at TRL. For simplicity, the approach adopted here is initially similar to that used by Jean[2] which only deals with a head-on impact into a solid immovable barrier.

Figure One.
Residual crush after a head-on barrier test



Since the collision is head-on, a fairly uniform crush profile should be expected. The US National Highway Traffic Safety Administration (NHTSA) make available all their crash test results. These are available through the WWW and also through the Accident Reconstruction Journal which publishes the results on a fairly regular basis. To produce more realistic examples, a sample test will be used. The results are published in miles per hour, weight in pounds and crush in inches so these been converted into metric units for this paper. The March / April 1995 edition of ARJ lists a Volvo 850 which was tested during 1994.

Vehicle	**Mass**	**Test Speed**	C_1	C_2	C_3	C_4	C_5	C_6	C_{ave}
Volvo 850	1442 kg	56.3 kmh^{-1}	41.9	45.7	48.8	49.5	46.5	43.1	46.0

Note: All crush measurements in centimetres. Width of front of vehicle 167 cm

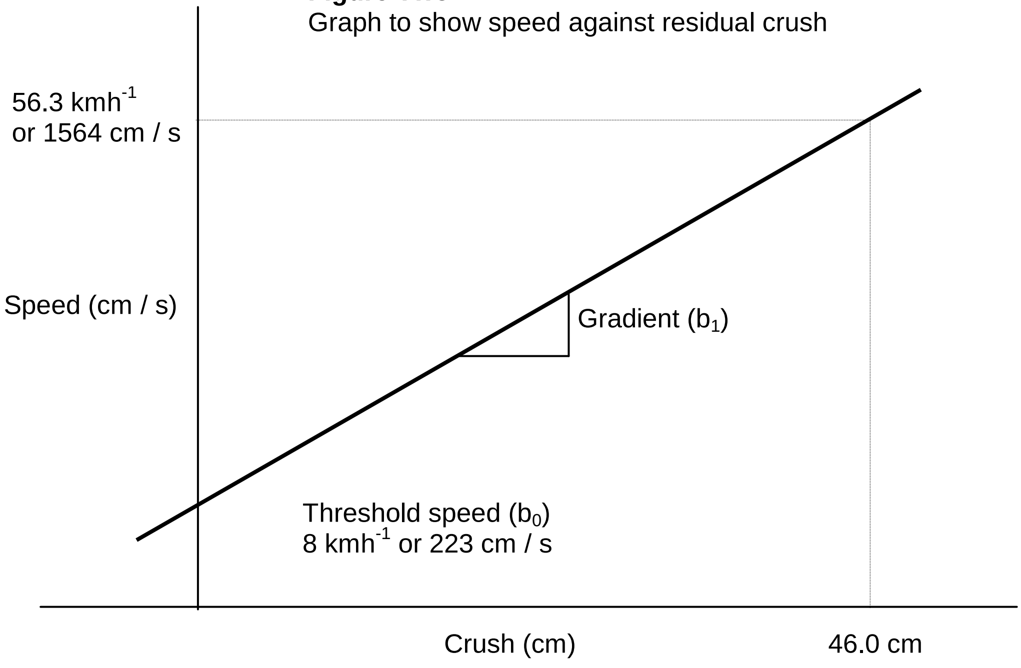
To simplify matters an average of all the crush measurements can be taken to produce just one value for crush against impact speed. In this example, the average crush was 46.0 cm for an impact speed of 56.3 kmh^{-1}.

Due to the design of vehicles impacts at very low speeds tend not to produce any residual crush. Rather the vehicle bumpers absorb the energy of impact and rebound to their original shape. It is important to have some estimate of this threshold speed. In the absence of a large data set with which to use statistics to generate the threshold speed, the only alternative is to guess. For this Volvo 850 a reasonable threshold speed is probably about 5 mph (8 kmh^{-1})

Speed / damage graphs

With the information acquired so far it is possible to produce a graph of speed against residual crush. This is shown in Figure Two.

Figure Two.
Graph to show speed against residual crush



Since this is a straight line graph, it is very easy to determine the equation of the line. With the data shown on the graph the equations in both sets of units become,

$$V_{cm/s} = 29.15\,C + 223 \qquad \text{and} \qquad V_{km/h} = 1.05\,C + 8$$

These equations are useful in their own right as they form an alternative description of the EBS mentioned previously. For example, if a Volvo 850 is involved in a head-on accident with a barrier and that the average crush depth is measured at 35 cm, the equations show that the initial speed of the Volvo was almost 45 kmh^{-1}. If the Volvo suffered the same damage due to a collision with another vehicle, it is not possible to calculate the initial speed. We cannot even find the change in velocity of the Volvo, since nothing is known about the energy absorbed by the other vehicle. What can be calculated however is the amount of energy absorbed by the Volvo due to the collision.

By deriving similar speed / damage equations for the second vehicle as well, the energy absorbed by this vehicle could also be determined. With knowledge of the masses of the vehicles the standard Delta-V equation, (A) can then be used to find the respective changes in velocity.

Force / crush graphs

Rather than go through this process for every collision to work out the energy absorbed, a more user friendly solution would be to derive a graph that showed energy against crush directly. In reality a graph of force against crush turns out to be even more useful.

The net effect of what we are trying to achieve is to convert the vertical (speed) axis of the graph in Figure Two into a force. Since force is a vector quantity we should really consider the speed to be a velocity by defining a direction. As a first stage we need to find a relationship between velocity and force. To do this consider the relationship both these quantities have with acceleration. Using calculus notation we can define acceleration in several standard ways,

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v\frac{dv}{dx}$$

v = velocity
x = displacement
t = time

The third variation is the most appropriate version for this purpose. The velocity is already defined and the expression dv/dt can be read as the rate of change of velocity with distance. This is exactly what the gradient (b_1) of our graph represents – the variation of velocity with crush distance. So we can rewrite the equation to give,

$$a = v\frac{dv}{dx} \quad = \quad v \times b_1$$

An expression for the velocity (v) already exists and is the equation of the graph in Figure Two which in generalised terms is given by,

$$v = b_0 + b_1 C$$

Substituting for v gives,

$$a = (b_0 + b_1 C)b_1 \quad = \quad b_0 b_1 + b_1^2 C$$

From Newton's Second Law and substituting for acceleration we obtain,

$$F = m(b_0 b_1 + b_1^2 C)$$

This only gives the total force if the whole of the front of the vehicle is deformed. It is more useful if we divide the above equation by the length of the damaged area L to give an expression for the amount of force per unit width of damage. If we do this and multiply out the brackets we obtain,

$$F = \frac{m}{L} b_0 b_1 + \frac{m}{L} b_1^2 C \qquad \textbf{(D)}$$

Our two coefficients, A and B, can now be defined as,

$$A = \frac{m}{L} b_0 b_1 \qquad B = \frac{m}{L} b_1^2$$

Substituting in equation D gives an expression for the force per unit crush of,

$$F = A + BC \qquad \textbf{(E)}$$

This enables us to draw our desired graph with the additional benefit that equation (E) is the equation of the line. This graph is shown in Figure Three. Before drawing the graph it is worth considering the fact that we really need the energy absorbed by the force *F* acting through the distance *C.* This is given by the area under the graph since the definition of energy is of a force acting through a distance to produce a displacement. In the notation used here this becomes, $E = FC$ With suitable annotation to show the three areas which are under the graph the graph become,

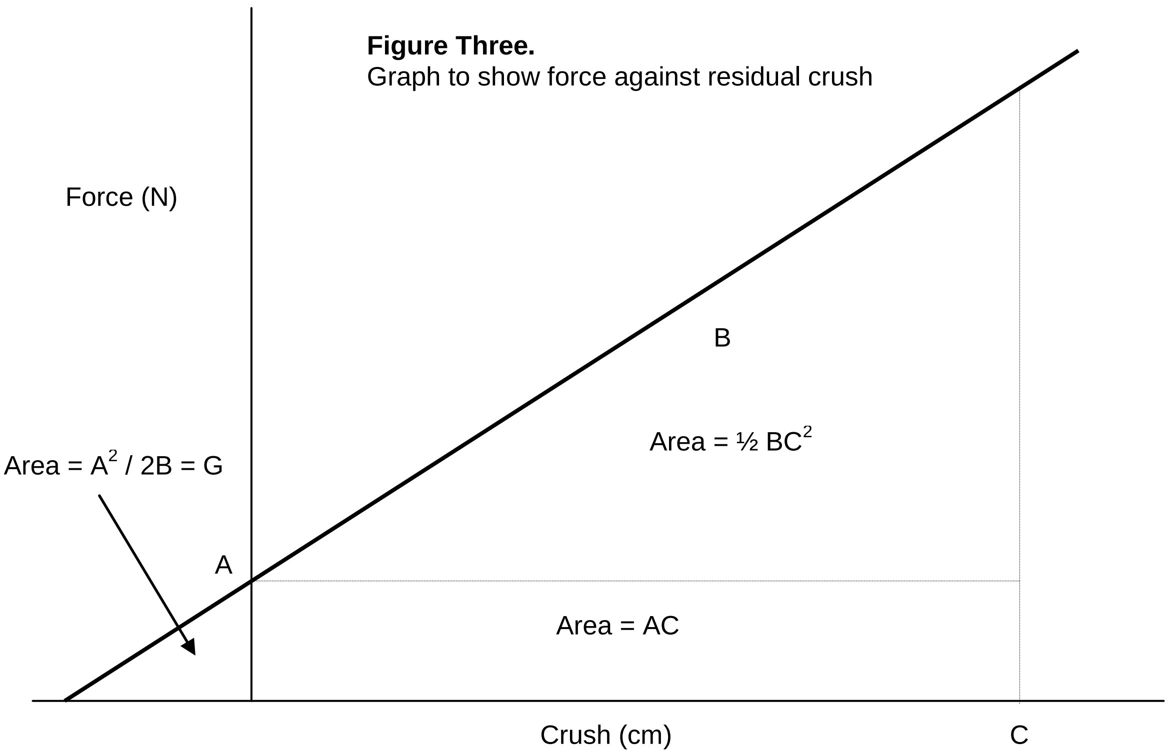


Figure Three.
Graph to show force against residual crush

By summing the three parts shown in Figure Three, the total area under the graph and therefore the energy per unit width can be found as,

$$E = AC + \frac{BC^2}{2} + \frac{A^2}{2B}$$

The total energy is therefore this expression multiplied by the total width of the damage giving,

$$E = L(AC + \frac{BC^2}{2} + \frac{A^2}{2B}) \qquad \textbf{(C)}$$

This is of course equation (C) as stated above without the additional term to allow for angled impacts. We can use this equation directly in calculations provided the crush damage is of constant depth, so that C is therefore a constant over the whole damage depth. Unfortunately this sort of damage is rarely seen in practice, so yet more modification is needed before we have an equation which can be used in all situations.

Another way of visualising the amount of crush caused to a vehicle is to consider a vehicle as consisting of a number of deformable springs. A force applied to these springs compresses them and due to the compression energy is absorbed. This also helps with considering the real nature of the A and B coefficients derived earlier. Figure Four shows one side of a vehicle as a series of these springs.

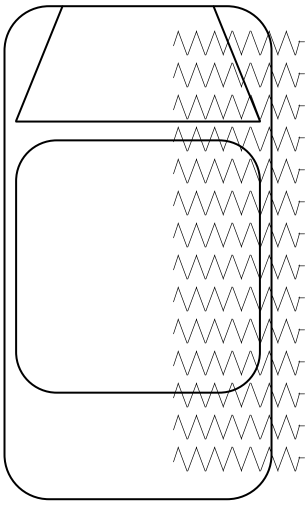


Figure Four.
Modelling the side of a vehicle as a series of springs

The stiffness of the springs is used in the programs as the *B* coefficient, a high stiffness implies that more force is required to compress them. Note also that the springs project beyond the outside of the vehicle. This highlights the fact that a certain amount of force is required before permanent deformation results. In equations (A) and (C) this is represented by the *A* coefficient.

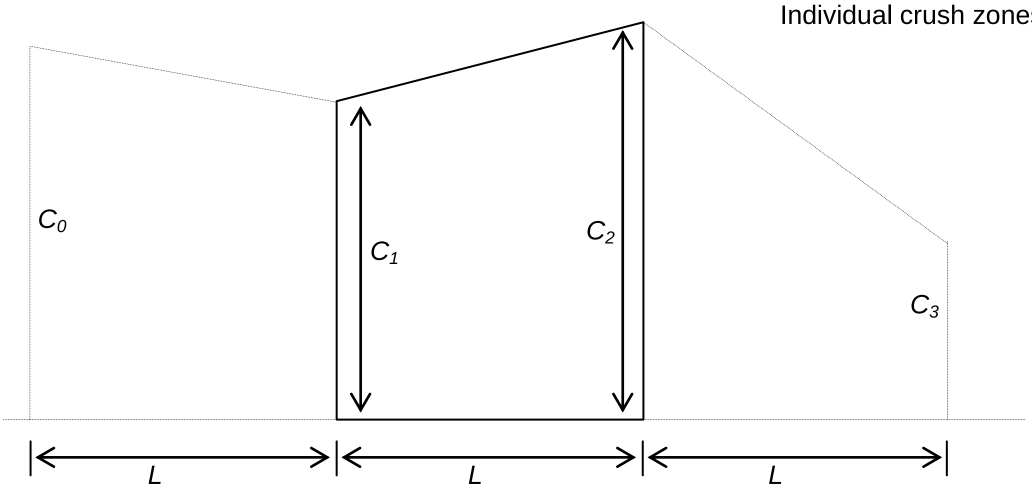
By taking this analogy one stage further, a physical meaning of the *G* coefficient can also be derived. Since a force is required to compress the springs through a distance, work is performed and therefore energy expended, as discussed earlier. The energy absorbed in crushing the springs just to the outside edge of the vehicle results in no deformation. If permanent deformation is caused, then this ‘pre-crush’ energy must have already been expended. For a particular set of coefficients this is a constant and is given by the *G* coefficient in the equations.

Individual crush zones
All damage profiles can be represented by a series of crush zones. For simplicity we show in Figure Five just one crush zone flanked by dotted lines indicating adjacent zones. The zone can be defined with two crush measurements, C_1 and C_2 and the width of the zone L. Without deriving the equation, which requires more advanced maths, we shall simply state the energy equation which represents the amount of damage absorbed by an individual crush zone.

$$E = LG + A \times area + B \times \bar{x} \times area \qquad \textbf{(F)}$$

L = width of zone
$\bar{x}$ = displacement of centre of mass of zone
area = area of zone

Figure Five.
Individual crush zones



Each crush zone is a quadrilateral and these can be considered as consisting of a rectangle and triangle. With this information the total area of a quadrilateral can be determined as,

$$area = \frac{L}{2}(C_1 + C_2)$$

The centre of mass of the quadrilateral is a little more difficult to derive but again a consideration of the separate triangle and rectangle which make up a quadrilateral help. The position of the centre of mass is given by the equation,

$$\bar{x} = \frac{C_1^2 + C_1C_2 + C_2^2}{3(C_1 + C_2)}$$

Once the area and position of the centre of mass have been determined, the energy absorbed can be calculated by substituting these into equation (F). The techniques described here for calculating the energy represented by one crush zone can be applied to any number of zones although it does get a bit tedious. Once this is done however the total energy absorbed by the damage profile is found as the sum of all the individual energies from each of the crush zones.

Example calculations
As an example we can take a head-on barrier collision, involving the Volvo 850 which results in conveniently shaped damage as shown in Figure Six.

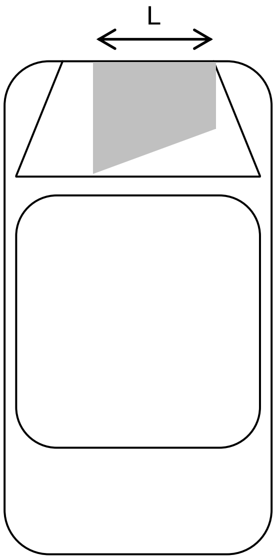


Figure Six.
Crush damage caused to Volvo 850.

C_1 = 35 cm
C_2 = 48 cm
L = 70 cm

The data obtained and the results of the various calculations are listed in the table below.

Data for Volvo 850 test vehicle		
	Value	**How obtained**
Mass	*1442 kg (14.42 Ns/cm)	Measured
Width of front	167 cm	Measured
Threshold speed (b_0)	223 cm / s	Estimated
Gradient of speed / damage graph (b_1)	29.15 s^{-1}	$b_1 = \frac{V_{Test} - b_0}{C_{ave}}$
A coefficient	561 N / cm	$A = \frac{m}{L} b_0 b_1$
B coefficient	73.4 N / cm^2	$B = \frac{m}{L} b_1^2$
G coefficient	2144 N	$G = \frac{A^2}{2B}$
Data for collision vehicle		
Mass	1200 kg	Measured
C_1	35 cm	Measured
C_2	48 cm	Measured
dL	70 cm	Measured
Area	2905 cm^2	$area = \frac{dL}{2}(C_1 + C_2)$
Position of centre of mass (Longitudinal Displacement)	21 cm	$\bar{x} = \frac{C_1^2 + C_1 C_2 + C_2^2}{3(C_1 + C_2)}$
Energy absorbed	** 61096 joules	$E = dLG + A \times area + B \times \bar{x} \times area$

* Since we are using centimetres, the appropriate unit of mass in these calculations is not measured in kilogrammes (which can be expressed as Ns/m) but in Ns/cm.
** Due to measurements in centimetres, we need to divide the values obtained by 100 to convert to the SI units of joules.

Note that equation (F) does not have any term involving the mass of the test vehicle. This is effectively eliminated from the energy calculation by the way that the *A* and *B* coefficients are calculated. We can and should therefore use the actual mass of the vehicles involved in the collision in the Delta-V equations.

We now have a value for the energy absorbed in the collision. It is a simple matter to insert the known values into equation (B) to obtain,

$$\Delta v = \sqrt{\frac{2 \times 61096}{1200}} \quad = 10 \text{ ms}^{-1} \text{ (36 kmh}^{-1}\text{)}$$

Let us now assume that the Volvo (vehicle one) is in collision with a Ford Escort (vehicle two) which has a mass of 1050 kg. The same pattern of damage emerges on the Volvo and similar calculations for the Escort reveal that the absorbed energy amounts to 84000 J. Note that there is no physical reason why the energy absorbed by each of the vehicles should be the same. By Newton's Third Law, the force acting between the two vehicles should be similar, but the energy is not so constrained.

The standard Delta-V equation (A) can be used to determine the change in velocity of the Volvo and the corresponding equation used for the Escort. This gives,

$$\Delta v_1 = \sqrt{\frac{2(61096 + 84000)}{1200(1 + \frac{1200}{1050})}} \quad = 10.62 \text{ ms}^{-1}$$

$$\Delta v_2 = \sqrt{\frac{2(61096 + 84000)}{1050(1 + \frac{1050}{1200})}} \quad = 12.14 \text{ ms}^{-1}$$

Of necessity these examples are fairly simplistic, but should serve to illustrate that it is possible to perform simple crush analyses using no more than a calculator. Other complications, such as energy magnification due to angled impacts, the effects of non-central impacts, or multiple crush zones have been ignored.

Problems with crash testing

Recent research by Neptune[3] suggests that some of the published crash test data may contain incomplete data. The cause of the problem is that some, more modern, vehicles are fitted with bumpers constructed of a rubber based material. Due to their construction the bumpers may rebound after impact to their original shape but importantly the underlying structure of the vehicle remains crushed.

This has been referred to as the 'air-gap' problem as the bumper creates a gap between the rebounded frontal face of the vehicle and the distorted vehicle behind.

The correct method for measuring crushed vehicles should involve measuring by pushing the rubber bumper inwards until the solid crushed structure is reached. Neptune reports that some of the crash testing facilities are allowing for the air-gap in this way, but that others may not.

The effect of incorrect measuring is to reduce the amount of crush recorded on the test vehicle. Since the impact speed is known this results in an increased *B* coefficient, assuming that the threshold speed remains constant. Overall this could result in overestimates of the impact speed where an investigator measures the vehicle correctly in the field.

This is of course an undesirable state of affairs and it is to be hoped that revised measuring procedures are implemented. It is only on the more modern vehicles that this phenomenon is noticed, so in reality this problem is unlikely to affect the validity of the crush coefficients used by either EDCRASH or Ai Damage. These coefficients are based on large data sets of crashed vehicles which were tested prior to 1992.

Care must be taken however if individual vehicles are used to generate crush coefficients for 'by-hand' calculations as described in this paper.

Partial overlap testing is now becoming popular, particularly since the New Car Assessment Programme (NCAP) began. This has given researchers the opportunity to test the CRASH3 model against partial overlap data. Neptune[3] reports that in his research, the CRASH3 model performs well.

Summary

Two methods of determining the change in velocity of a vehicle involved in a collision were explained. Both methods require crash test data. A simple linear equation to determine the EBS for head-on collisions was proposed and this was extended to develop the coefficients used in the CRASH3 derivative programs.

An example of how to use the crash test data to derive the change in velocity for a simple two vehicle collision was presented. The model presented is incomplete in that it does not allow for angled impacts, energy magnification and the effects of non-central impacts.

The air-gap problem was discussed together with the potential effects.

References

[1] **Neades, J** *Using and Understanding AiDamage* AiTS training document, 2003

[2] **Jean, Brian** *Calculating Stiffness Coefficients from Barrier Test Data*. Accident Reconstruction Journal Vol. 4 No. 5, 1992

[3] **Neptune, James A.** *Comparison of Crush Stiffness Characteristics from Partial Overlap and Full-Overlap Frontal Crash Tests*. SAE 1999-01-0105, 1999