SWERVES AND LANE CHANGES

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INTRODUCTION

Lane change and swerve manoeuvres occur in many practical situations. A vehicle may need to change lane in order to perform an overtake of another vehicle, or may simply swerve in an attempt to avoid a collision. Accident investigators only tend only to become involved when there has been a collision, and this manoeuvre has not been completed. Investigation into these types of manoeuvres, relates mainly to considering whether or not the attempted swerve or lane change could and should have been completed in safety.

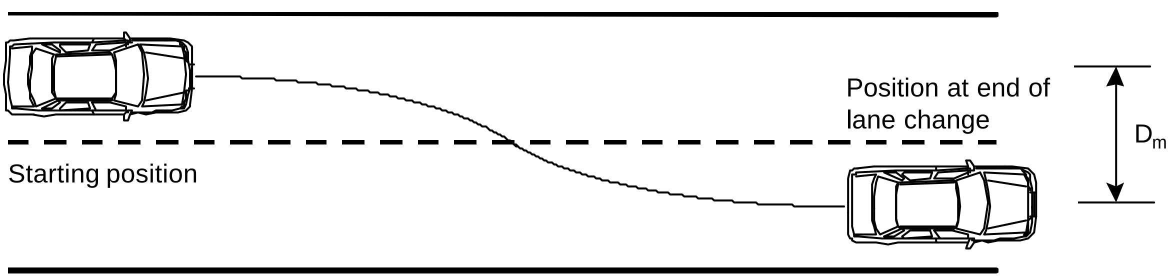
In some situations we may need to find the time it took for a vehicle to perform a swerve or lane change. In other circumstances we may need to determine the distance over which this manoeuvre takes place. For example when answering questions such as 'Could the driver have regained his own side of the road in the distance available?'

Throughout this article we will deal with the *minimum* times and distances needed to perform a swerve or lane change manoeuvre. We do not attempt to estimate how long it takes for a driver to apply maximum steer in one direction and then change the steering to the opposite direction. This all takes time and in that time the vehicle will travel along the road.

We explain how it is possible to use the concept of critical speed to generate additional information about the behaviour of the vehicle. We look firstly at the time needed to perform a lane change and then consider the distance needed to complete the manoeuvre.

TIME TO COMPLETE A SWERVE

A useful starting point for our discussion on swerves and lane changes is to consider a complete lane change. The type of scenario we are considering is shown in the diagram below,



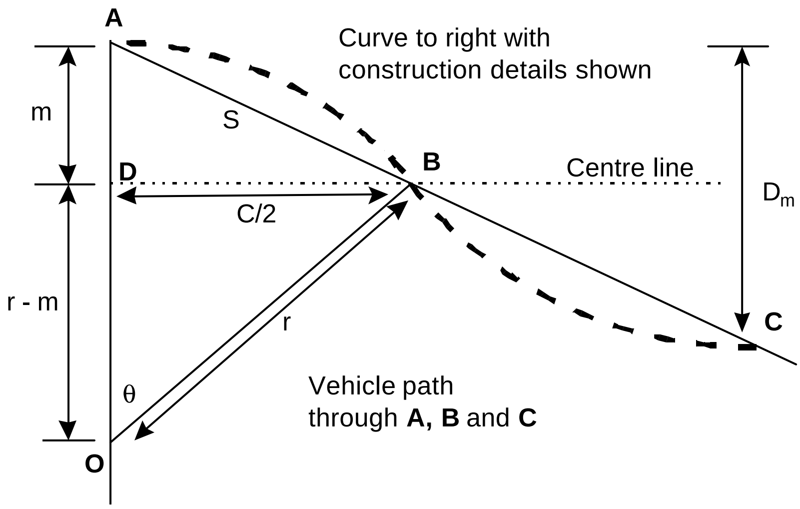
D_m = lateral distance through which centre of mass has moved

The vehicle starts in one lane and moves into the next lane. If we consider the motion of the centre of mass we can see from the diagram that it moves laterally through a distance which we call D_m (Distance moved).

If the vehicle is travelling at a given speed (V) we can find the total time needed to perform the complete manoeuvre. The way in which we find the total time is to consider the path taken by the centre of mass to be two separate turns. The first, in this example, is to the right and the second to the left.

From an understanding of critical speed, we know that if we can find the radius of the turn, we can find the theoretical maximum speed at which that turn can be negotiated. Once the speed at which the turn can be taken is known, we can calculate the time needed to complete the turn relatively easily. This is the procedure that we follow to find the time taken.

Our next step is to redraw the diagram above with a little more information and labels needed to explain various features. We have included information needed to calculate the radius of the turn on the diagram below.



As previously stated, the turn can be considered to be two turns. Our technique for deriving an equation is based on that described by Jennings[1] to find the time for one of these curves and then double the result at the end to find the time for the complete manoeuvre.

Consider the curve from A to B. We can see that it is an arc with a half the chord shown (C/2) together with the mid-ordinate measurement (m). The distance OA and OB are radii (r) of the circle we are trying to find.

It is helpful at this stage to consider what information we have available. We only actually know the mid-ordinate (m). This is equal to half the total lateral displacement (D_m). This is not quite the whole story as we can find, from other methods, the coefficient of friction (μ) and we always know the value of g at 9.81 ms^{-2}.

We could find the true distance from A to B using the equation,

$$S = 2\pi r \left(\frac{\mathrm{Cos}^{-1}\,\theta}{360} \right)$$

Where $\cos^{-1}\theta$ is the inverse cosine of the angle theta and $\cos\theta$ is given using trigonometry as,

$$\cos\theta = \frac{r - m}{r}$$

Continuing in this way leads to some tedious calculations. To make life easier we can make an approximation and state that the distance A to B is in fact a straight line. Although not providing a perfect answer this makes our derivation very much easier. In addition this approximation becomes more realistic as the radius (r) increases.

Calculation shows that at 13 ms^{-1} with a coefficient of 0.7 and a lateral deviation of 10 m the error this approximation makes in the final result is about 1.8%. At a higher speed of 31ms^{-1} this error is reduced to 0.3%. Lower values of mu or smaller lateral deviations also reduce the error.

Consider the triangle OBD, this is a right angled triangle. Using Pythagoras' Theorem we can write an expression for half the chord length (C/2) in terms of the radius (r) and the mid-ordinate (m),

$$\frac{C}{2} = \sqrt{r^2 - (r - m)^2}$$

The triangle ADB is also a right angled triangle, using our simplification above, and Pythagoras again we can derive an expression for S,

$$S = \sqrt{\left(C/2\right)^2 + m^2}$$

Since we already have an expression for C/2 in equation (A), we can substitute this into equation (B) to give,

$$S = \sqrt{r^2 - (r - m)^2 + m^2}$$

This simplifies quite nicely to become,

$$S = \sqrt{2\ r\ m}$$

We already know the equation for critical speed,

$$v = \sqrt{r\,\mu\,g}$$

We also know that the average change of distance in given by the equation,

$$S = V\,t$$

We can substitute our critical speed equation (v) for V in the average speed equation to give,

$$S = t\sqrt{r\,\mu\,g}$$

Equations (C) and (D) are both equal to the distance S. We can therefore equate them to give,

$$\sqrt{2\,r\,m} = t\,\sqrt{r\,\mu\,g}$$

This can be solved for t to give,

$$t = \sqrt{\frac{2\,m}{\mu\,g}}$$

This is only the time for half the manoeuvre. We need to double this to find the time for the whole manoeuvre. We can also make the substitution,

$$D_m = 2m$$

The minimum time needed to perform a complete lane change manoeuvre is therefore given by the equation,

$$t = \sqrt{\frac{4\,D_m}{\mu\,g}}$$

The only factors affecting the time are the distance through which the vehicle moves laterally, D_m and the coefficient of friction (μ). This equation does not contain any term involving the speed of the vehicle. This is an important point. It means that the minimum time needed to perform a lane change manoeuvre is totally independent of the speed. Whether travelling at 5 mph or 105 mph, the minimum time needed to change lanes is a constant.

DISTANCE TO COMPLETE A SWERVE

In the previous section we derived an expression for the minimum time needed to complete a lane change manoeuvre. Let us consider again, what information we have available. We still know the lateral distance through which the vehicle moves (D_m) and now we have the time (t) as well.

If we can estimate the speed at which the vehicle is moving, we can calculate the distance over which the vehicle travels using the equation,

$$S = V\,t$$

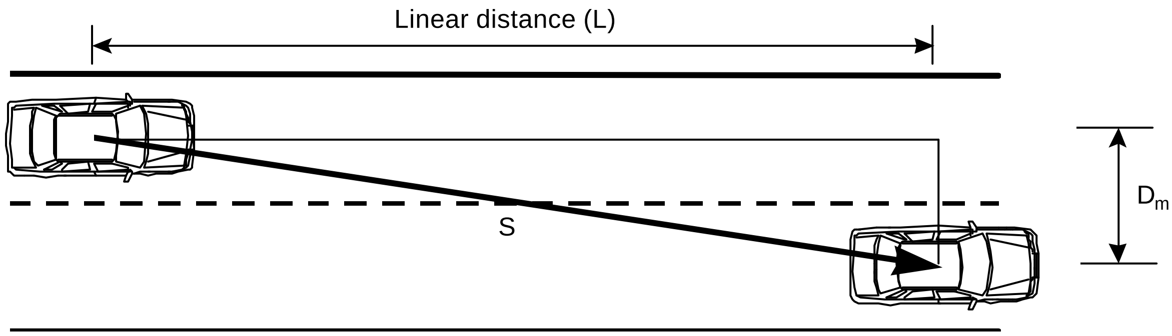
We can substitute our expression for time, as found previously, into this equation to give,

$$S = V\sqrt{\frac{4\,D_m}{\mu\,g}}$$

We can move the term for speed (V) inside the square root sign to give,

$$S = \sqrt{\frac{4\,V^2\,D_m}{\mu\,g}}$$

Already we have a relatively simple expression for the distance over which the vehicle travels in performing a lane change manoeuvre. However this is not the linear distance along the road but the diagonal distance through which the vehicle travels, according to our straight line approximation made earlier. This distance (S) and the linear distance (L) are as shown in the diagram below.



We can use Pythagoras' Theorem to determine the linear distance (L), because S is the hypotenuse of a right angled triangle. So we have,

$$L = \sqrt{\frac{4\ V^2\ D_m}{\mu\ g} - D_m^2}$$

At high speeds the D_m^2 term is negligible in comparison to the other term under the square root and can often be ignored.

Worked Example

A vehicle is travelling at a constant speed of 120 kph on a motorway. The driver sees a stationary car in the carriageway ahead. To avoid a collision the driver must swerve laterally 10 m to the right. The coefficient of friction is 0.80. How far will the vehicle travel along the road in changing lanes to the right, and how long will this take?

We can substitute the given values directly into the equation derived above,

$$L = \sqrt{\frac{4\ V^2\ D_m}{\mu\ g} - D_m^2}$$

Which gives,

$$L = \sqrt{\frac{4 \times (120 \div 3.6)^2 \times 10}{0.8 \times 9.81} - 10^2}$$

$$= 74.59 \text{ m}$$

NOTE: If we had ignored the lateral change of width D_m in this equation we would obtain the result 75.25 m. This represents a difference of less than 0.9 % over the more accurate calculation.

We need also to find the time (t) during which the manoeuvre took place. Using the equation,

$$t = \sqrt{\frac{4\,D_m}{\mu\,g}}$$

We can substitute the given values to obtain,

$$t = \sqrt{\frac{4 \times 10}{0.8 \times 9.81}} = 2.3\ \text{s}$$

Comment

These equations can be varied to suit a number of different situations. For a single swerve rather than a complete lane change, we can represent the manoeuvre by just one curve. The derivation earlier was based on two swerves.

Using similar arguments, we can derive expressions for the time taken and distance covered for a single swerve. Using corresponding notation, we obtain,

$$t = \sqrt{\frac{D_m}{\mu\,g}} \qquad L = \sqrt{\frac{V^2\,D_m}{\mu\,g} - D_m^2}$$