# Skid testing with confidence

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**Abstract**

Determining a suitable coefficient of friction is fundamental for many reconstructions and this parameter is often estimated by skid testing[1]. The uncertainty associated with the range of values produced by any testing is also important and can be quantified using statistical techniques to establish intervals based on a desired level of confidence. Suitable techniques to establish such confidence intervals are presented. Current practice within the UK is to perform two skid tests to fall within 10% of the lower value and frequently to utilise the lower coefficient in order to produce a minimum speed for a vehicle. The minimum speed of a vehicle is often the most favourable value for a defendant in criminal cases, but this is not always the case and in civil cases a probable speed is often preferable which can be derived from the mean coefficient of friction. It is demonstrated that using the results of two tests produces a coefficient of friction which is accurate to a 50% level of confidence. It is shown that performing three or more skid tests produces a significantly higher level of confidence. It is also shown that for any specific level of confidence, the confidence interval for coefficients of friction is reasonably small by ensuring that the skid tests all fall within 10% of the minimum test value. Smaller confidence intervals can be generated if the skid test results fall within a smaller range. It is suggested therefore that a 'minimum three test' rule is adopted by all investigators in order to increase the statistical significance of any skid testing. Such tests should also lie within 10% and preferably 5% of the lowest value.

**Introduction**

Many collision reconstructions require an investigator to determine the coefficient of friction (drag factor) between various surfaces. The coefficient of friction between tyres and the road is most frequently required, but coefficients between tyres and other surfaces may also be required as too might be the coefficient applicable to a sliding motorcycle. The comments in this article apply equally to other forms of testing, however for convenience this discussion concentrates on determining the coefficient of friction for tyres sliding over road surfaces using skid tests. Generally skid testing involves causing a vehicle to slide over the surface and then either measuring the acceleration using an accelerometer (e.g. Vericom, SkidMan) from which the coefficient of friction can be estimated, or by skidding to a stop from a known initial speed and measuring the skidded displacement. If the latter method is used, then an estimate of the coefficient of friction can be obtained using the well-known formula

$$\mu = \frac{u^2}{2gs} \tag{1}$$

where $\mu$ is the required coefficient of friction, $u$ is the initial speed, $g$ is the acceleration due to gravity and $s$ is the skidded displacement. Further details about the origin of equation (1) and skid testing techniques are available in the AiTS notes [**1**] or other publications e.g. [**2**], [**3**]. Adjustments to an estimate of the coefficient of friction can also be made to take into account other factors such as gradients or incomplete wheel lockup [**4**].

Whatever method is used, it is general practice in the UK for a police investigator to perform two skid tests and if the coefficients of friction are within 10% of each other, use the lower value in subsequent calculations. Whether using the lowest value is necessarily the best to use in a particular collision scenario is discussed later. In civil court reconstructions the use of the mean or average value is more likely to be used. If the results are not within 10% then additional tests are

[1] Skid testing is a general term used to denote tests carried whilst under emergency braking consideration.

performed. It is noted that other countries use different criteria for skid testing. Australia and New Zealand police for example perform at least three tests to determine the coefficients of friction their investigators use in calculations. Again the requirement is that all tests should produce results that are within 10% of each other.

The discussion here considers the question of whether the UK practice of performing two tests is sufficient to produce a reliable coefficient of friction and what level of confidence can be assigned to the results of skid testing. Using simple statistical methods, the current UK situation is considered first followed by a discussion of the effect of increasing the number of tests.

**Statistical analysis**

In this section a statistical analysis of skid test results is performed to determine the likely range of results suggested by the data. This is explained using an example series of tests and considering the results. It is assumed that two suitable tests were performed yielding coefficients of friction between the tyres and the road surface of 0.71 and 0.73. It is obvious that these are well within 10% of each other, so are likely to be accepted by a UK investigator. The lower value of 0.71 is then likely to be used by the police in calculations to determine the speed of a vehicle. For example, if a vehicle left 25 metres of locked wheel marks when skidding to a stop, the initial speed of the vehicle causing the marks can be calculated to be 18.65 $ms^{-1}$ using the familiar equation

$$u = \sqrt{2\mu g s}. \qquad (2)$$

This speed of 18.65 $ms^{-1}$ may well be quoted as the 'absolute minimum speed' with the justification that it utilises the lowest coefficient of friction and does not take into account any pre-skidding loss of speed. Whether this speed does actually represent the likely minimum speed needs to be considered.

Statistical methods allow data such as skid test results to be analysed to produce a range in which the true mean value is likely to fall. This is known as a confidence interval and provides a range of values which is likely to include the unknown true mean value. The extent of this range depends on the level of confidence required by the investigator. A higher level of confidence produces a larger range of values and a smaller confidence level produces a smaller range. The confidence level is the probability $(1-\alpha)$ associated with a confidence interval. Confidence levels are often expressed as a percentage so that for example if $\alpha = 0.05$ then the confidence level is equal to $1-0.05 = 0.95$, i.e. 95%. A confidence interval of 95% is often used although 90% and 99% are also commonly used.

The techniques necessary to determine confidence intervals for single sample tests are explained by Neades [**5**] and in many books on elementary statistics such as Elements of Statistics [**6**]. A brief summary of the method and equations necessary to establish a confidence interval are described below. The mean value for a set of data $(\bar{x})$ can be determined from

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad (3)$$

where $n$ is the number of data items. Another quantity known as the standard deviation $(\sigma)$ is also required. The standard deviation is a measure of the dispersion of the data, in other words how much the set of data deviates from the mean or average value. For a sample of $n$ items, the standard deviation can be determined using the equation

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}. \qquad (4)$$

Using equations (3) and (4) the mean of the two coefficients of friction is found to be 0.72 with a standard deviation of 0.014. A confidence interval $(CI)$ can then be calculated using the equation

$$CI = \bar{x} \pm CV \times \frac{\sigma}{\sqrt{n}} \tag{5}$$

where *CV* is a critical value dependent upon the confidence interval required.

In this example the data set consists of just two values. For small data sets a set of statistical tables known as Student's t distribution are required to provide the critical value. To obtain a value the number of degrees of freedom is required and this is determined simply as the number of individual values *n* minus 1 i.e. $df = n-1$. With only two tests the number of degrees of freedom for this analysis is one. An example of a suitable table from Statistics for Collision Investigators [**5**] showing just the first five rows is shown below with the 95% confidence levels highlighted. Note that for this analysis an investigator has no reason to suppose the actual coefficient of friction is either higher or lower than the mean value so what is known as a 2-tailed value is required. In other applications a 1-tailed test may be more appropriate.

**Table 1: First five rows of Student's t-table**

| 2-tail α | 0.5 | 0.2 | 0.1 | 0.05 | 0.02 | 0.01 | 0.001 |
|---|---|---|---|---|---|---|---|
| **Confidence** | **50%** | **80%** | **90%** | **95.00%** | **98%** | **99.00%** | **99.90%** |
| **1** | 1.0000 | 3.0777 | 6.3138 | 12.7062 | 31.8205 | 63.6567 | 636.6192 |
| **2** | 0.8165 | 1.8856 | 2.9200 | 4.3027 | 6.9646 | 9.9248 | 31.5991 |
| **3** | 0.7649 | 1.6377 | 2.3534 | 3.1824 | 4.5407 | 5.8409 | 12.9240 |
| **4** | 0.7407 | 1.5332 | 2.1318 | 2.7764 | 3.7469 | 4.6041 | 8.6103 |
| **5** | 0.7267 | 1.4759 | 2.0150 | 2.5706 | 3.3649 | 4.0321 | 6.8688 |

For one degree of freedom, the critical value is found to be 12.706. Using Microsoft's Excel program, the TINV(*α*, *df*) function can also be used to find the equivalent value where *α* is the confidence required (0.05 in this example) and *df* is the required number of degrees of freedom. The 95% confidence interval for the likely value of the true mean can now be calculated

$$\begin{aligned} CI &= 0.72 \pm 12.706 \times \frac{0.014}{\sqrt{2}} \\ CI &= 0.72 \pm 0.13 \\ CI &= \{0.59,\ 0.85\} \end{aligned} \tag{6}$$

For a 95% level of confidence this analysis shows that the coefficient of friction might actually fall anywhere between 0.59 and 0.85 Assuming that the lowest speed is desired then using 0.59 as the lowest value in equation (2) now produces a speed for the vehicle of 17 $ms^{-1}$ which is approaching 10% slower than the originally calculated speed. This is a significant reduction in what was originally determined to be the minimum speed determined from two tests that were relatively close. In general, larger differences between the two original skid test values produce correspondingly larger confidence intervals for the likely mean value. For example if skid test results of 0.71 and 0.77 were obtained, these are still within the UK accepted tolerance of 10%, however a 95% confidence interval for the true mean is found to be the very large interval of {0.36, 1.1}.

From inspection of the input data to equation (5), the source of the large confidence interval is clearly the presence of the large critical value of 12.706. The solution to reducing the confidence interval is to reduce this large critical value. There are two ways in which this can be achieved. The first option is to accept a lower confidence level and the second option is to perform more tests. Adopting the first option, to reduce the confidence interval so that the lower bound matches the original 0.71 lowest test value, the confidence level must be reduced to 50%. This indicates too that the confidence level achievable from performing just two tests is also 50%.

The effect of performing one more skid test is now examined So that the mean value remains constant, it is assumed that the third skid test produces a coefficient of friction of 0.72. Thus the mean value remains the same but the standard deviation reduces to 0.01. The confidence interval can now be calculated in the same way, but now with two degrees of freedom so the critical value reduces significantly to 4.303 The confidence interval can now be calculated using equation (5) as {0.70, 0.74}. Using the earlier example, a 25 metre skid to stop now suggests a range of initial speeds of between 18.52 to 19.04 $ms^{-1}$. If these represented real-world results and in the absence of uncertainty in the other data it can be argued that to a 95% level of confidence the speed of the vehicle was within the range 18.52 to 19.04 $ms^{-1}$.

It is also worth considering also the effect of a third skid test on the more diverse results of 0.71 and 0.77 where the 95% confidence interval was found to be unrealistically large at {0.36, 1.1} Performing one more skid test which for simplicity is also assumed to fall on the mean value of 0.74, now produces a 95% confidence interval of a more realistic {0.67, 0.81}.

As a result of this analysis it is suggested that investigators consider the use of a third skid test to make their choice of a coefficient of friction more robust. To ensure statistical reliability and to generate a reasonably small confidence interval it is also suggested that all three tests should all fall within 10% and preferably 5% of the lower value.

Further analysis shows that if a 10% range is maintained then the 95% upper and lower bounds for the confidence interval will lie within no more than about ±12% of the mean value. For example if a typical mean value of 0.75 for three tests are found, all of which fall within 10% of the lower value, then the 95% confidence interval is {0.66, 0.84}. If the range of skid test results is reduced to 5% then the 95% confidence interval is reduced {0.705, 0.795} and the upper and lower bounds of the confidence interval are no more than ±6% of the mean value.

In general, if the skid tests form a tighter range than 10% of the lower value then the confidence intervals decrease accordingly. It should be noted however that in order for the 95% confidence interval to fall within the same 10% or 5% range of skid tests, more tests are needed. If both the upper and lower bounds are to be matched then the middle value needs to lie on the mean value as well. Between five to seven tests are required to ensure the lower bound of the confidence interval is greater than the lowest skid test value obtained and this depends on the distribution of test results within the range. To ensure that the lower bound of the confidence interval matches the lowest skid test result using only three tests, the confidence level must be reduced. It is found that the confidence level required to match the lower bound with the lowest skid test result from performing three tests is in the region of 58% to 82%. If the middle test falls on the mean value, the confidence level is 77%. Although less than a 95% confidence level, this represents a marked improvement over the 50% confidence level achievable with just two tests.

**Discussion**

Regardless of how many skid tests are performed, the question of the coefficient of friction to be utilised in any subsequent calculations should also be considered. In general a criminal investigation is primarily concerned with the question of whether a person may have committed a traffic offence. Criminal courts tend to adopt the principle '*in dubio pro reo'* where any doubt is interpreted to the favour of a defendant. In practice this often means that criminal investigations tend to be focused on determining minimum speeds and to this end criminal investigators generally utilise the lowest coefficient of friction as discussed previously.

It should be emphasised that using the lowest coefficient of friction from two or three physical tests does not produce a minimum speed to a 95% level of confidence. To do that requires use of the lower bound from a confidence interval.

There are also circumstances where producing the minimum speed is not to the benefit of a defendant. For example, consider a scenario where a vehicle turns across the path of an oncoming vehicle. If the oncoming vehicle was within sight of the turning driver, then it seems likely that the turning driver would be culpable. However, if the oncoming vehicle were out of sight of the turning driver when the turn commenced, perhaps because of the speed of the oncoming vehicle, their liability becomes less certain. In such circumstances it may be of more benefit to the defendant to use the highest coefficient of friction to produce higher speeds for the oncoming vehicle. Other collision scenarios may also suggest that a higher coefficient of friction might be of benefit to a defendant so each case should be considered on its merits.

In the civil courts the emphasis is somewhat different. Here the most probable speeds are of far more relevance and these are the speeds that are normally calculated. As such it is usual that the mean coefficient of friction is used in order to produce the 'most likely' speeds. With both sorts of investigation however, it is desirable that an investigator should be able to quantify the likely range of results from any testing that has been performed and assign confidence levels to that range.

## Conclusions

In general, if an investigator wishes to establish a range of likely coefficients of friction from skid tests, this can be determined to the desired level of confidence using the techniques described here. If one coefficient of friction is to be utilised in calculations, then the investigator should consider whether this should be chosen from the lower or higher end of the interval or whether using the mean value is more appropriate.

The current UK practice of performing two skid tests generates coefficients of friction that indicate a confidence level of 50%. This can be improved to around 77% if three tests are performed. It is suggested therefore that three tests are performed to determine the appropriate coefficient of friction. These tests should be within 10% of each other and preferably 5% in order to generate a reasonably small confidence or credible interval.

Using the lowest physical test result to determine the speeds of a vehicle does not produce the 'absolute minimum speed' for that vehicle. Instead it produces a minimum speed to some level of confidence which can be calculated. To determine the minimum speed of a vehicle to a specific level of confidence, a confidence interval must be established and the lower bound of that confidence interval utilised in calculations.

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