Road Accident Investigation

The phrases 'the police service' and 'using mathematics' are not usually associated with each other. There are however a small number of police officers and other specialists in the UK who use mathematics to reconstruct the probable manoeuvres of vehicles involved in accidents. It is often possible to show how fast vehicles were travelling prior to impact, and provide information about causes of the accident. This article explains a few of the principles behind road accident investigation and how they are applied.

It has long been recognised that marks on the road surface, damage and final positions of vehicles can give an indication as to what actually happened during an accident. It is only within the last 30 years or so that any real attempt has been made at putting this on a scientific footing. Modern accident investigation originated in the United States during the late 1960's in a attempt to analyse an increasing number of injury accidents and to develop safety mechanisms for road vehicles. It was introduced into the United Kingdom a few years later for the same reasons.

Most of the techniques used are based upon the definitions of acceleration and velocity together with Newton's three laws of motion. Using these simple definitions a whole wealth of equations and methods can be derived. They are reproduced below,

Velocity - the rate of change of distance with time.

Acceleration - the rate of change of velocity with time.

Newton's First Law - A body remains at rest or continues at uniform velocity in a straight line unless acted on by an external force.

Newton's Second Law - The rate of change of momentum is proportional to the applied force and takes place in the direction of that force.

Newton's Third Law - To every action there is an equal and opposite reaction.

One of the advantages of dealing with vehicles in critical situations, such as accidents, is that drivers tend to react positively (provided they do actually react). This leads in turn to marks being left on the road, either through excessive braking or steering. As an example let us take an ordinary car skidding along a road and show how locked wheel marks are produced and what information can be gained from an examination of those marks.

Skidding vehicles leave marks on the road surface, or indeed any other surface across which they are moving. These marks are caused by the tyres interacting with the road surface. All of us will have seen such marks at some stage. They are invariably black in colour and the usual assumption is that the mark is merely rubber deposited on the road surface. This may be true on a concrete surface where only light marks are seen, but on a asphalt road surface, the dark black marks are caused mainly by bitumen.

The first few metres of the marks will be composed of tyre rubber but as the tyre skids across the surface it naturally heats up due to friction. In fact it heats up to such an extent that the bitumen within the road melts and 'bleeds' through to the surface leaving black lines.

To an accident investigator these marks tell a story about which tyre on the vehicle left a particular mark, the condition of the tyre and the attitude of the vehicle making the mark. Clues about the make and model of vehicle and a good approximation of the initial speed of the vehicle can also be ascertained.

A vehicle skidding along a road will experience a transfer of weight, about the centre of mass, to the front wheels reducing the weight at the rear wheels. This causes the front tyres to deform raising the centre of the tyre off the road surface. The net effect is to produce two parallel marks along the road made by the outside edges of the tyre.

The rear tyres undergo the reverse effect. The inside of the mark is quite distinct, but the outside edges less well defined. The set of tyres making the mark can therefore be identified easily. This effect is more pronounced with older style diagonal ply tyres but is still visible using radial tyres. These effects are illustrated in Diagram 1.

Diagram 1: Front and rear tyres showing the effect of weight transfer and marks produced.

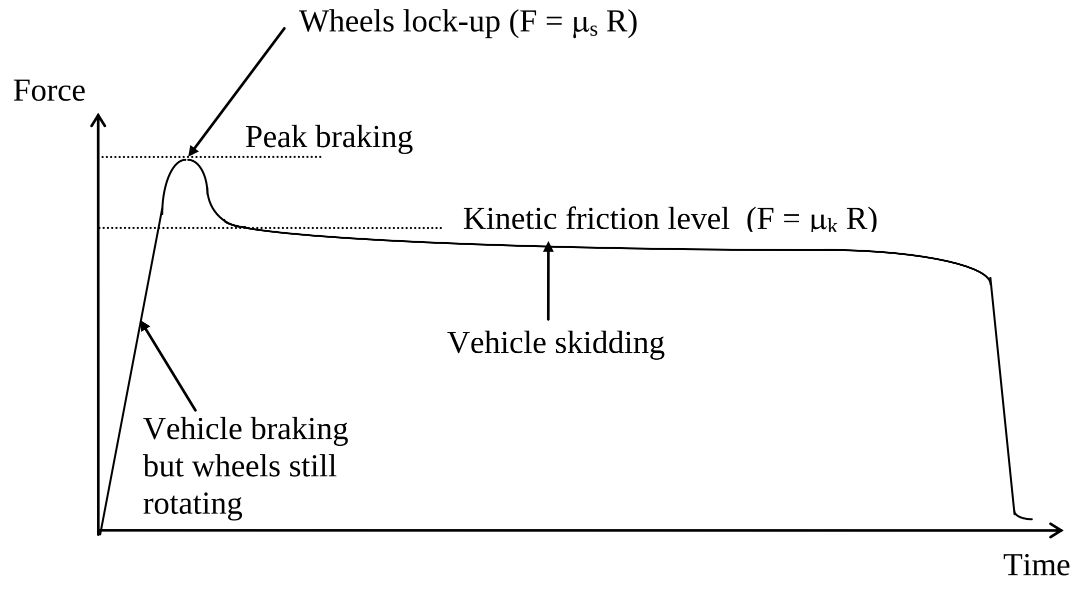
Front tyre with concave deformation

Rear tyre with convex deformation

The track between the front and rear sets of marks can be measured and if the final resting point can be identified, the wheelbase can also be measured. In accidents where a vehicle has failed to stop, this can provide important information about the missing vehicle.

It is helpful to consider exactly what happens as the driver applies the brakes in an emergency situation. The graph in Diagram 2 shows a generalised curve of the variation of braking force with time and illustrates some important features.

Diagram 2: Graph showing variation of force with time during braking and subsequent skidding



There is a sharp rise to maximum braking as the driver applies the brakes. As the wheel locks the force produced drops slightly and is followed by a lengthy period where the force produced is more or less constant. This corresponds to the time while the vehicle is skidding. The vehicle therefore slows at a roughly constant rate, until it comes to a halt.

Prior to a wheel locking a point of maximum braking efficiency is reached. This corresponds to the stage where the wheel is at the point of locking. The frictional force (F) produced can be shown to be equal to the product of the normal reaction of the vehicle (R) and the coefficient of friction (μ),

$$F = \mu R \qquad (1)$$

The frictional force produced is limited by the coefficients of friction. Until the wheel locks the maximum force is governed by the static coefficient. Once sliding however, the kinetic or sliding coefficient limits the force and dominates any calculations due to the greater time for which it is acting . The kinetic coefficient of friction is always lower than the static coefficient. This can be demonstrated by considering the force required to start an object moving compared with the force needed to keep the object moving. The initial force will be higher.

If the rate of deceleration over the main part of the skid can be found then it is possible to estimate the initial speed. It is relatively straightforward to perform skid tests from a known speed and calculate the coefficient of friction between the tyre and the road surface.

Using the definitions of average velocity and acceleration, an equation can be derived connecting the initial (u) and final (v) velocities together with the skidded displacement (S), coefficient of kinetic friction (μ) and the acceleration due to gravity (g),

$$v^2 = u^2 - 2\mu g S \qquad (2)$$

A consideration of Newton's Second Law shows that the frictional force is proportional to the mass of the vehicle. So a more massive vehicle generates more force than a smaller vehicle, as it slides along the road. Both however decelerate at the same rate, determined by the same coefficient of friction.

Once the coefficient of kinetic friction for the test vehicle has been found, an estimate of the speed of the accident vehicle from the length of the locked wheel marks can be calculated. (See box) There are several other methods of determining this value but these are not discussed further.

This procedure makes a number of assumptions and simplifications. Primarily it is assumed that the tyres on the accident vehicle and the test vehicle yield the same coefficients of friction on the same road surface.

Tests show that vehicle tyres can be divided into three main categories, based upon the vehicle to which they are fitted. These categories are motorcycles, cars and heavy commercial vehicles. Within each category the frictional coefficients of individual tyres on the same surface have been found to exhibit only a small variance, typically less than 5%. Between the categories however the difference is much greater. Any variations could be alleviated by using the accident vehicle in a test skid, but this is not always possible due to collision damage.

All the wheels of both the accident vehicle and the test vehicle need to be locked or operating at near maximum efficiency during the skidding for the results to be anything like accurate. The simplification made here is that all the reaction due to the weight of the vehicle is contributing to the production of the frictional force. Obviously if only some of the wheels are contributing to the braking effort then adjustments need to be made.

With modern vehicles, brake limiter valves are very effective in preventing the rear wheels from locking before the front. This safety feature assists in the stability of the vehicle under braking.

Unfortunately they can be so effective that rear wheels do not always lock up, even during emergency braking, so no marks are left on the road surface. Unless an examination of the vehicle shows that a rear wheel could not have contributed to the braking effort the assumption is made that it was contributing fully.

If examination shows a defect, such as contamination with brake fluid, it is possible to calculate how much braking would have been provided by the vehicle. This procedure involves finding the height of the centre of mass so that moments about this point can be used to establish the weight transfer to the front. The effect of gradients and unbraked trailers are modelled in a similar manner by considering the forces acting upon the vehicle. From both methods an ‘effective’ coefficient of friction can be derived and more realistic calculations performed.

The biggest problem with modern vehicles, from an investigators point of view, is the increasing use of anti-lock braking systems (ABS). A common misconception is that ABS allows a vehicle to stop in a shorter distance. In certain conditions this may be true, but anti-lock systems are designed to prevent the wheels of a vehicle locking which allows for some steering during emergency braking. Whether the driver is capable of braking and steering in an emergency situation is quite another matter. No locked wheels mean no locked wheel marks on the road surface. This is not particularly helpful in establishing initial speeds.

Most ABS systems are designed to allow wheels to reach a point where they are nearly locked, corresponding to just before the peak in Diagram 2. The brakes then release slightly to allow further rotation before they are reapplied. The whole cycle is repeated many times a second. Well designed systems operate so close to full lock-up that the tyre is rotating slower than it is travelling across the road. This can produce faint marks on a road. These marks are usually composed of tyre rubber and are equivalent to the early stages of a full locked wheel mark. They may be intermittent, due to the on / off nature of the system and have a tendency to disappear within a short time.

Despite another common belief, the depth of tread on a tyre has no effect on the stopping qualities of that tyre on dry roads. The only effect of tread on a tyre is to disperse water in wet conditions. Lack of tread can be important in determining aquaplaning speeds but that is another topic.

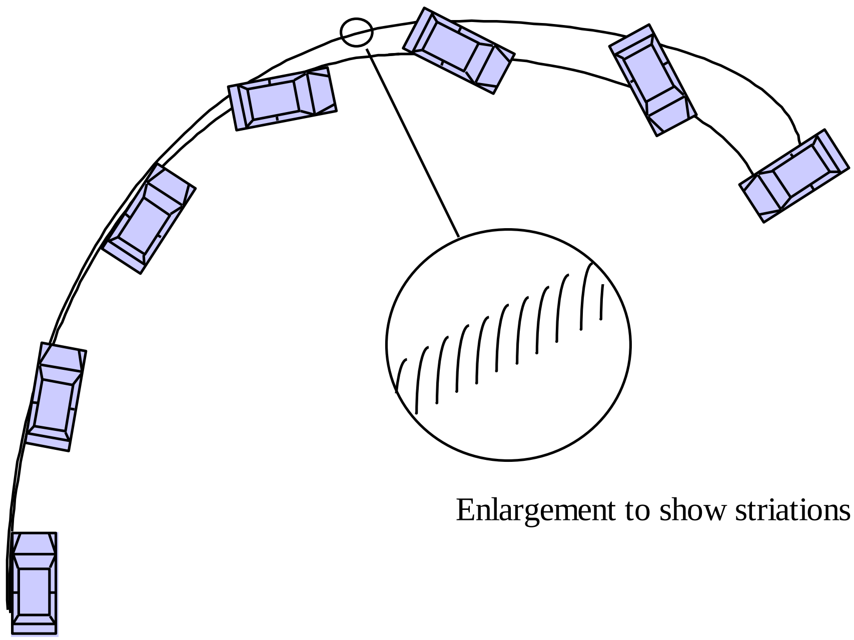
Accident investigation is not limited to vehicles moving in a straight line. The behaviour of vehicles whilst cornering is a fascinating subject. Potentially a number of variables could be considered, such as lateral and longitudinal weight transfer, suspension effects, tyre dynamics, and a whole host of other factors. In most situations a reasonable approximation is reached if the vehicle is treated simply as a particle in circular motion with variations applied for any road camber.

The maximum lateral force that can be generated by a tyre is predominantly governed by the coefficient of friction (μ), and the normal reaction in much the same way as discussed previously. It is possible to derive an expression for the force required to cause a vehicle to accelerate towards the centre of a circle. Equating these two forces generates an equation for the maximum speed (V) at which a vehicle can negotiate a bend of known radius (r),

$$V = \sqrt{r\mu g} \qquad (3)$$

Situations can arise where a vehicle has attempted to negotiate a bend at too high a speed. Inevitably the vehicle describes an arc of greater radius than the bend, generally with the vehicle drifting sideways out of control. Apart from feeling uncomfortable for the occupants, a vehicle in this condition tends to leave characteristic striated scuff marks on the road surface which follow a roughly circular path. Diagram 3 shows the general behaviour of a car in this 'critical speed' situation.

Diagram 3: Vehicle in critical speed showing the tyre marks produced



During collisions objects can be projected from vehicles. A consideration of the motion of these objects under the influence of gravity can give estimates as to the speed of the launch vehicle. This has been applied to pedestrian accidents and it is possible to calculate vehicle speeds from the distance a pedestrian is thrown.

There are several methods in common use involving the principle of the conservation of linear momentum to establish changes in velocity due to impact. For simplicity, a two dimensional analysis using orthogonal momentum vectors is performed. Three dimensional analyses may be more accurate but the necessary data is not often available.

A more advanced variation, including this principle, is to estimate the energy required to deform a vehicle and thereby calculate changes in velocity. Perhaps the greatest difficulty with this particular method is in finding a good approximation of the energy required. Computer packages exist which assist in establishing these figures. Research is continuing by the several institutions to find more realistic estimates of the energy of deformation.

Computers have been used by investigators for some time, particularly to assist in the more tedious calculations. It seems inevitable that the general trend towards increased use of technology will affect the accident investigation industry, hopefully for the better. Specialist accident investigation tools and surveying software are now readily available.

Some products allow an investigator to define vehicle parameters, give some initial conditions and watch as the programs display what would happen. These are powerful tools, but with so much control over the program input, it is essential that the operator has a detailed understanding of the algorithms used. Without such knowledge any results must be questionable.

New road and vehicle safety features are introduced all the time. The use of seat belts in both the front and rear seats has already proven its worth. Within the last five years ABS braking systems have become common. Traction control systems, air bags, side impact protection systems and a variety of other measures are currently fashionable. Whether or not those features do actually make motoring safer is open to debate and continually appraised.

It has not been possible to give more than a flavour of the blend of maths, physics and engineering principles that comprise accident investigation. It is not an exact science by any means, but is composed of a variety of approximations and assumptions which if handled correctly yield good results.

Finally, if the reader is unfortunate enough to become involved in an accident, spare a thought for the investigator and see if their conclusions match your recollections of the accident - you may be surprised.

Further reading:

The Principles of Accident Investigation, Jon Neades & Ric Ward (1995) ISBN 1 900452 00 6

Vehicle Handling Dynamics, J. R. Ellis (1994) ISBN 0 85298 885 0

About the author:

Jon Neades is a director of AiTS, a training and software company specialising in accident investigation. Until recently he was a police sergeant holding the post of Head of the Law & Accident Investigation Department of the Regional Police Driving School in Devizes, where he still lectures on a regular basis. He is a mathematics graduate of the Open University and has taught accident investigation as far afield as Central America. He is currently involved in developing accident investigation software and related training products.

Example

Finding the speed of a vehicle from the skid length

In a real reconstruction an investigator would determine the actual coefficient of friction between the tyres and the road surface. Using this coefficient, together with the length of locked wheel marks in equation 2 means that the speed of a vehicle can be determined.

In practice, car tyres on a dry road, usually produces a coefficient of friction of between 0.65 to 0.8. If we further assume that the car skids to a stop, we can rewrite equation 2 to yield maximum and minimum speeds in miles per hour (u) from a skid length in metres (S).

$$u_{min} = 8 \times \sqrt{S} \qquad u_{max} = 8.9 \times \sqrt{S}$$

With a skid length of 15 m for example, these equations show that a car was travelling at a speed of 31 to 34 mph when it started to slide.