Determining The Maximum Speed At Which A Bend Can Be Negotiated

Jon Neades
AiTS, Aerial View, Shab Hill, Birdlip, Glos.

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Abstract
Techniques for establishing the actual speed of vehicles from striated tyre scuff marks are well known. In this paper a method is discussed which considers the inverse problem; what is the maximum speed at which a particular bend can be negotiated? The method presented demonstrates that the maximum speed at which a bend can be negotiated is dependent upon finding the maximum radius that a vehicle can take through a particular bend. That maximum radius is itself determined by the inside radius defining the bend itself, the available road width and the angle through which the bend turns. This technique is limited to bends which have a constant radius and presents a method for establishing the maximum radius of turn for any particular bend. An analysis of the dependency between the maximum radius the angle of turn and available width is also provided.

Notation

c	chord length
m	middle ordinate
r	radius
v	speed of the vehicle
g	acceleration due to gravity
μ	coefficient of friction
W	available width
θ	angle of turn
R_{max}	maximum radius
V_{max}	maximum speed
b	subscript used to identify base value
max	subscript used to identify maximum value

Introduction
The technique to establish the speed of a vehicle which leave striated scuff marks is well known and is described by Smith [1991][1]. It can be shown that the if the radius of the outer front tyre mark can be determined over the early part of the mark, then the speed of the car marking the marks is established. The radius of the mark can determined from a chord and middle ordinate measurement using the expression,

$$r = \frac{c^2}{8m} + \frac{m}{2} \qquad (1)$$

Using computer surveying software other methods can be utilised for establishing the radius of the marks from any three distinct, non-collinear points. Naturally, particular care should be taken in ensuring that the surveyed points which are to be used in this way are accurately surveyed.

Once the radius is established, the speed of the vehicle making the marks can be determined from the expression,

$$v = \sqrt{r\mu g} \qquad (2)$$

Equation 2 is often described as the critical speed equation. Lambourn [1989][2] describes the origin of the striations and considers the accuracy of this method. He concludes that in all the tests he performed this expression produced results accurate to ±10%. Lambourn also suggests guidelines which should be applied to the use of striated tyre marks to estimate speed so that the ±10% accuracy limit can be satisfied. (These guidelines are reproduced in Appendix A to this paper.)

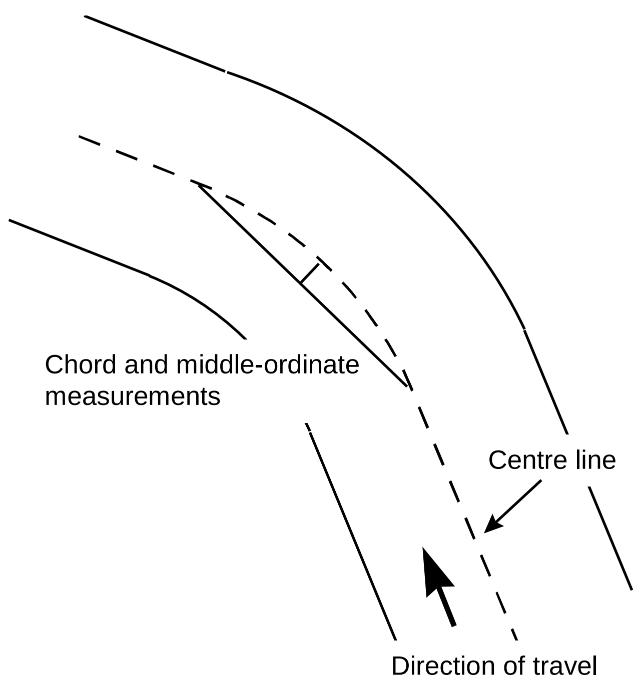
Hague et al [1997][3] performed another series of tests to determine whether the ±10% accuracy suggested by Lambourn was applicable to more modern vehicles. In two tests they calculated an underestimate slightly in excess of 10% however the remaining tests were all within ±10%.

Derivation

In this paper the inverse problem is considered. Given a particular bend, what is the maximum speed at which the bend can it be negotiated? Neades [1991][4] considered the same problem. This paper expands on that that earlier work and demonstrates a new derivation. Additional analysis of the results are also considered in this paper.

One way to find a radius of the bend is immediately apparent. The radius of the bend can be established using a chord and middle ordinate. Together with the coefficient of friction, the maximum speed possible around the bend can then be established using the critical speed equation (Equation 2). Figure 1 below shows a typical bend

Figure1. Simple method to establish radius of centre line of road



As an example, we can put some data into the diagram above. We find that for a chord length of 35 m we obtain a middle ordinate of 800 mm. Let us assume, in the absence of other data, that the coefficient of friction is 0.70. The aim is to find the maximum speed for a vehicle which is to remain on its own side of the road while negotiating this left hand bend. The radius of the central white line is given by equation 1.

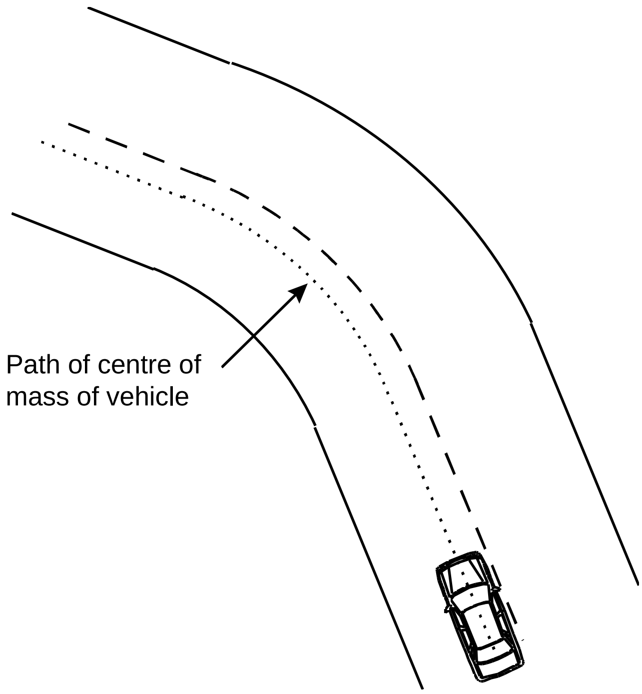
Substituting known values we obtain,

$$r_b = \frac{35^2}{8 \times 0.8} + \frac{0.8}{2}$$

r_b = 191.81 m

The largest radius that the centre mass of the vehicle follows while negotiating this left hand bend, is just inside the white line as shown in figure 2. Note that this diagram assumes that drivers normally drive on the left hand side of the road.

Figure 2. Path of vehicle around bend – following centre line



The dotted line shows the path of the centre of mass of the vehicle. The centre of mass can be approximated by the mid-point of the vehicle. In other words the radius of the path is half the width of the vehicle inside the central white line.

Since the central white line and the path of the centre of mass are concentric circles, the radius of the path of the vehicle is simply the radius of the central white line, less half the width of the vehicle. If the width of the vehicle is 1.6 m the radius of the path of the centre of mass is,

$$191.81 - 0.8 = 191.01 \text{ m}$$

Using the critical speed equation (equation 2), the known values can be substituted to give,

$$v_b = \sqrt{191.01 \times 0.70 \times 9.8}$$

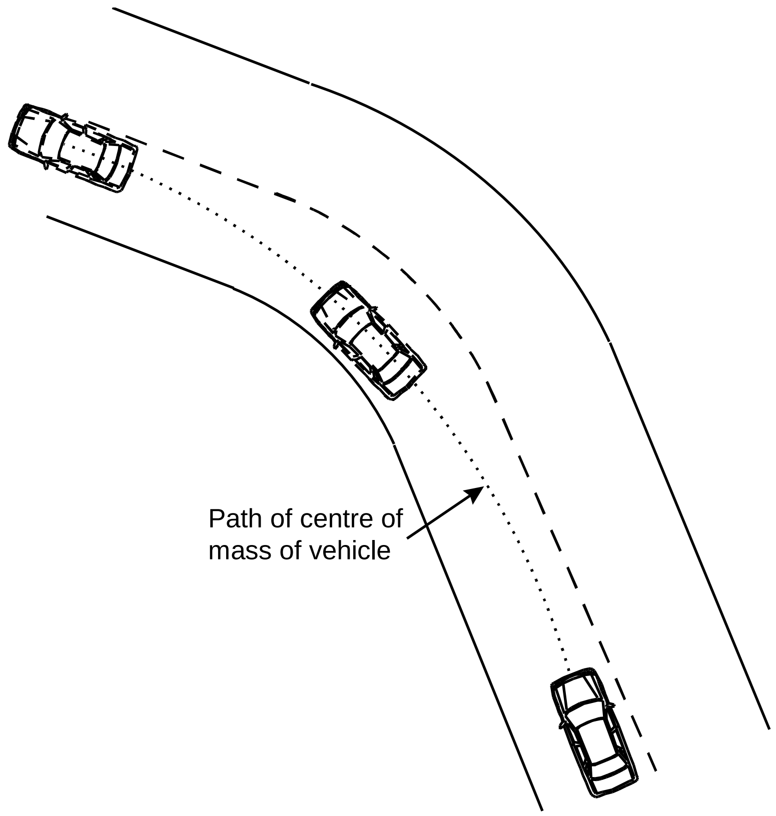
$$v_b = 36.20 \text{ ms}^{-1} \quad \text{(81 mph, 130 kph)}$$

Although this speed is traditionally quoted by investigators as the maximum speed at which the bend can be negotiated, it is suggested that this cannot be correct. It seems probable that a driver may attempt to ‘straighten’ the curve as much as possible so that the circular path followed by the vehicle is somewhat larger than calculated above. If a larger radius is followed around the bend, then clearly the bend can be negotiated at a higher speed.

Such a path through the bend is often described as ‘taking the racing line’ or ‘straightening the bend’. Essentially it involves the driver anticipating the bend and positioning their vehicle towards the outside of the bend on the approach. The driver then negotiates the bend allowing the vehicle to move towards the inside of the bend, coming close to the apex and then allowing the vehicle to move back towards the outside of the bend on the exit.

Such a path causes the vehicle to follow a larger radius than the actual radius of the bend and is shown in figure 3.

Figure 3. Path of vehicle around bend – largest radius R_{max}



The radius of the path followed by the centre of mass is again shown on the diagram using a dotted line. This radius may be considerably larger than the traditional estimate. It starts before the bend itself begins, close to the centre line marking, approaches the inside radius of the kerb, and then moves out towards the central line again some distance after the bend ends. What is required is a method for establishing the radius of the circular path followed by the vehicle around this bend.

The method described here accounts for vehicles which do take this optimal through a bend and is the maximum possible radius that a vehicle could follow. Any maximum speed calculated from this radius must therefore be the true maximum speed at which a bend can be negotiated.

Experience suggests that a bend which does not turn through a large angle can be negotiated at a higher speed than a much sharper bend of the same radius. (At the limit, a straight road which can be considered as a bend with a zero angle of turn, which can theoretically be negotiated at an infinite speed.)

In addition the available width of roadway is a consideration. A wider width in which to manoeuvre the vehicle from the outside to the inside should result in a bend which can be negotiated at a higher speed.

In order to calculate this maximum radius additional information about the angle through which the bend turns and the width of the lane or road will therefore be required. These are both factors which will affect the final result. Both these measurements are relatively easy to obtain and will be discussed in detail later.

In this paper a geometrical argument is followed to describe how an expression may be derived to determine the maximum radius. The full derivation is presented in Appendix B and leads to equation 3.

$$R_{\max} = r + \frac{W}{1 - \cos\left(\frac{\theta}{2}\right)} \qquad (3)$$

Equation 3 is very useful when attempting to determine the maximum theoretical speed for a bend. It does however make a number of simplifying assumptions which do limit its use. The equation for R_{max} cannot deal with a bend which changes radius around the turn. Such bends are often described as 'tightening up' which means that the radius of the bend reduces as the bend is negotiated.

The derivation makes no allowance for the effects of changing crossfall. Provided the crossfall remains constant across the width of road this equation can be used. Where the crossfall changes significantly, the R_{max} equation will be of limited use. In comparison, the simple approximation used at the beginning of this paper does not deal with changing crossfall either. Of the two methods described, in general the R_{max} equation will probably provide the more accurate answer.

Measurement of Variables

To simplify the written form of equation 3 two terms are used which require further explanation.

The actual road width in which the vehicle can move is determined by the investigator. The investigator will need to decide whether the vehicle is permitted to use the whole road or is constrained to remain within its own lane. Obviously if constrained to the width to just the one lane, the calculated value for R_{max} will be reduced. The speed subsequently calculated should not be described as the maximum speed for the bend but as the maximum speed for the bend, if the vehicle is to remain in its own lane.

Figures 4 and 5 show how the available width *W* and inside radius *r* are measured.

Figure 4. Measurement of Available Width (*W*)

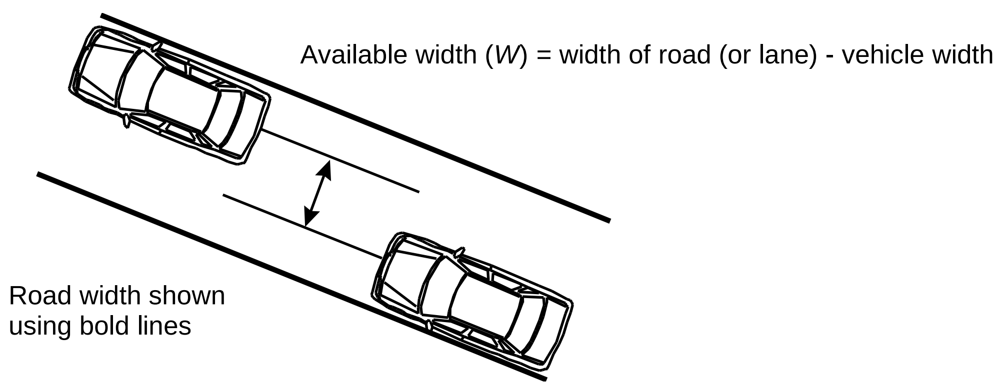
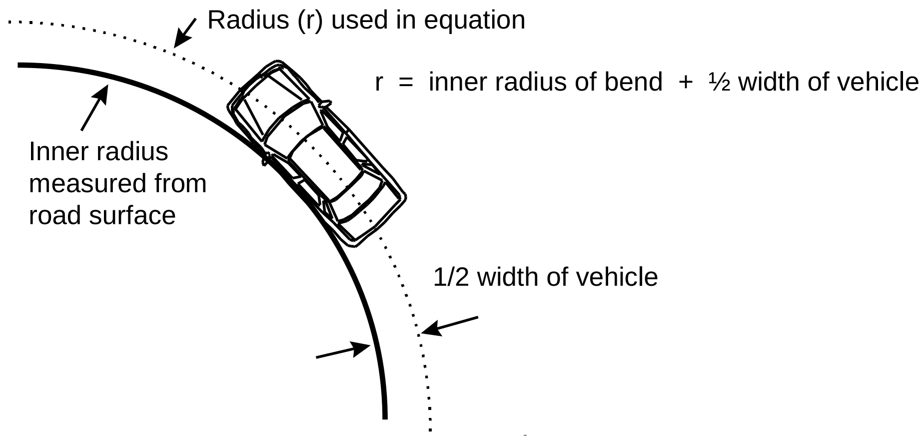


Figure 5. Measurement of Inside Radius (*r*)



Note that (*r*) is **not** the inside radius of the bend itself, but the inside radius that the path of the centre of mass follows around the bend that is used in the derivation of the equation. The 'closest' that the centre of mass can get to the inside of the bend is half the width of the vehicle, hence the addition of the half vehicle width to the inside radius. In summary, the available width in which the vehicle can move is given by *W* and the inside radius is given by *r* where

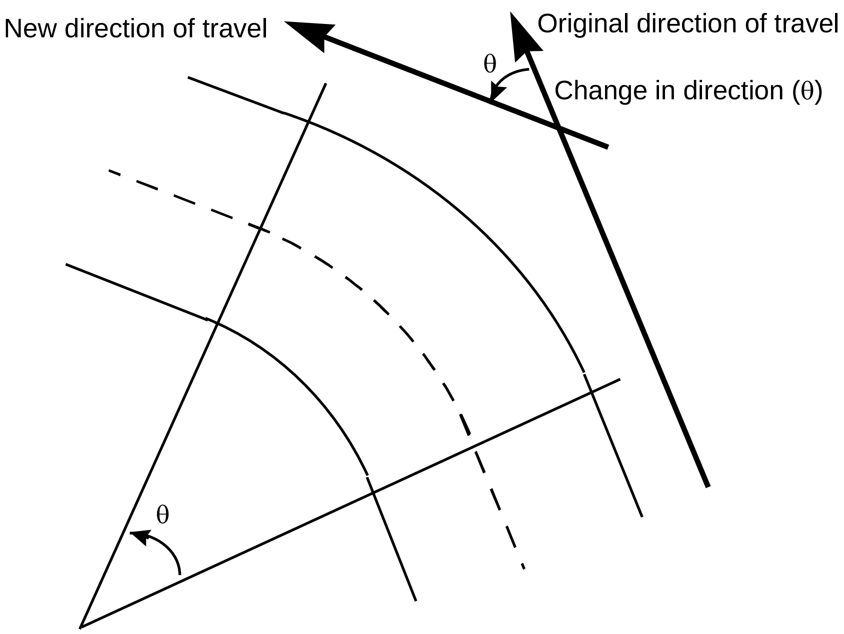
$$W = \text{Road (or lane) width - width of vehicle} \quad (4)$$

$$r = \text{inner radius of bend} + \tfrac{1}{2}\text{ width of vehicle} \quad (5)$$

In practice both the available width (*W*) and the radius (*r*) are straightforward to find when applying equations 4 and 5.

In practical situations, the only remaining difficulty in using the equation is in finding the value of angle of turn, theta (θ). This angle can often be measured directly off a surveyed plan, or from a suitable large scale national maps. The measurement of the angle of turn is shown in Figure 7.

Figure 6. Measurement of Angle of Turn (θ)



Discussion

To show how much difference a consideration of the angle of turn and the available road width can make to maximum speed calculations, the example used earlier may be used. In that situation the vehicle was shown negotiating a left hand bend. The vehicle was 1.6 m wide and the radius of the centre white line was calculated to be 191.01 m.

For the purposes of this discussion we can further assume that the lane width is 3 m.

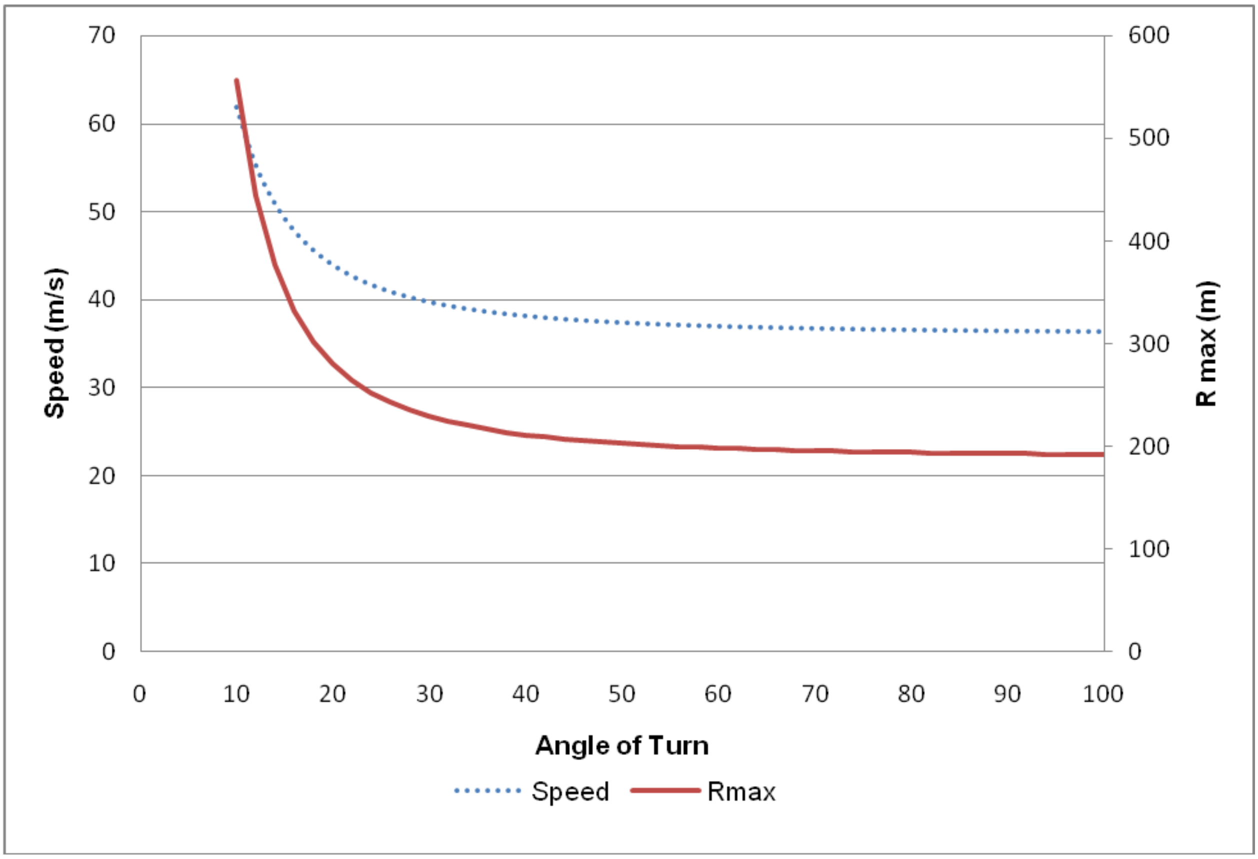
To find the inside radius of the curve itself the lane width of 3 m must be subtracted giving us 188.01 m. The inside radius (r) and available width (W) used in the R_{max} equation are given using equations 4 and 5 as follows,

$$r = 188.01 + \tfrac{1}{2}(1.6) = 188.81 \text{ m}$$

$$W = 3 - 1.6 = 1.4 \text{ m}$$

Substituting these values in the R_{max} equation (Equation 3) for various angles of turn (θ) we obtain the following results,

Graph 1. Variation of Maximum Radius and Speed with Angle of Turn.

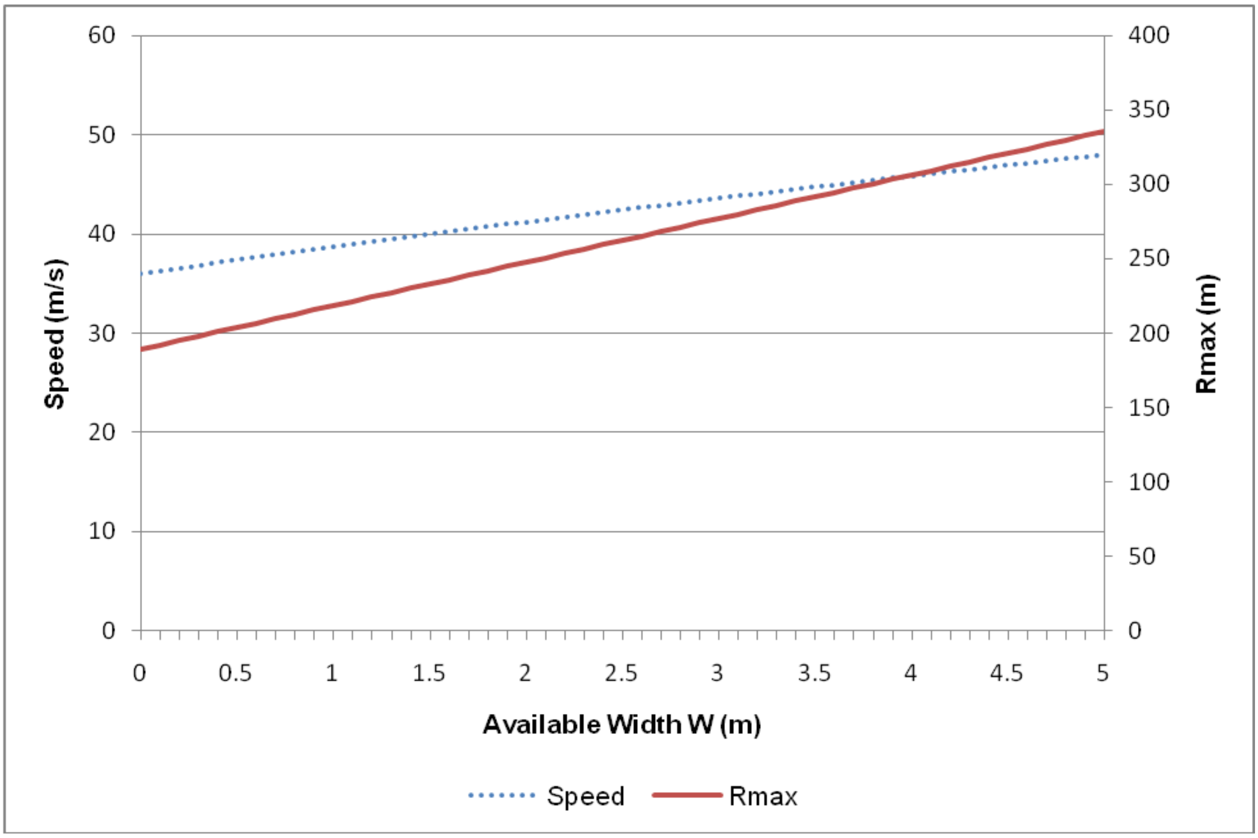


Compared with the initial constant value of v_b obtained earlier of 36.20 ms^{-1} it can be seen that at small angles of turn there is an very significant difference. At larger angles of turn the difference is smaller.

Continuing this graph to 180° shows that at 180° the calculated value for R_{max} is identical to the initial constant value V_b. Examination of equation 3 reveals that approaching 180°, the term cos(θ/2) tends towards zero. Similarly where there is a very small angle of turn, the maximum radius tends upwards towards infinity as cos(θ/2) tends towards unity, leading to the denominator tending towards zero.

A similar graph can be constructed to show how the available road width *W* affects results. In this example the angle of turn is fixed at 30°.

Graph 2. Variation of Maximum Radius and Speed with Available Width.



The general trend shown by Graph 2 is for the maximum radius and therefore the maximum speed to increase with an increase in available width. As the available width tends towards zero, the fractional term in equation 3 also tends to zero resulting in the maximum radius becoming closer to the inside radius *r*.

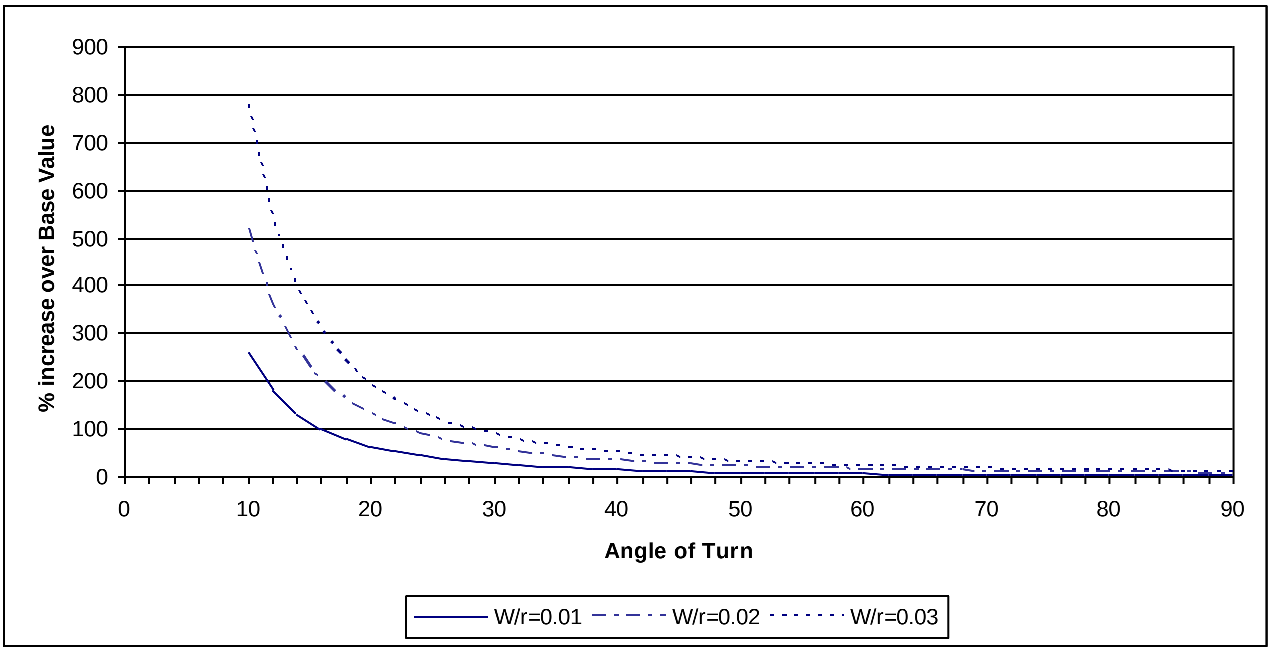
It is helpful to attempt to generalise these results for all bends in order to demonstrate how the maximum radius, angle of turn and available width are related. This allows some classification of a bend to determine whether the variation in maximum radius or speed is likely to be significant.

To make any comparison meaningful, the variation in angle of turn and available width can be compared with a constant base value (R_b). This constant base value is simply the radius determined using the traditional method discussed previously.

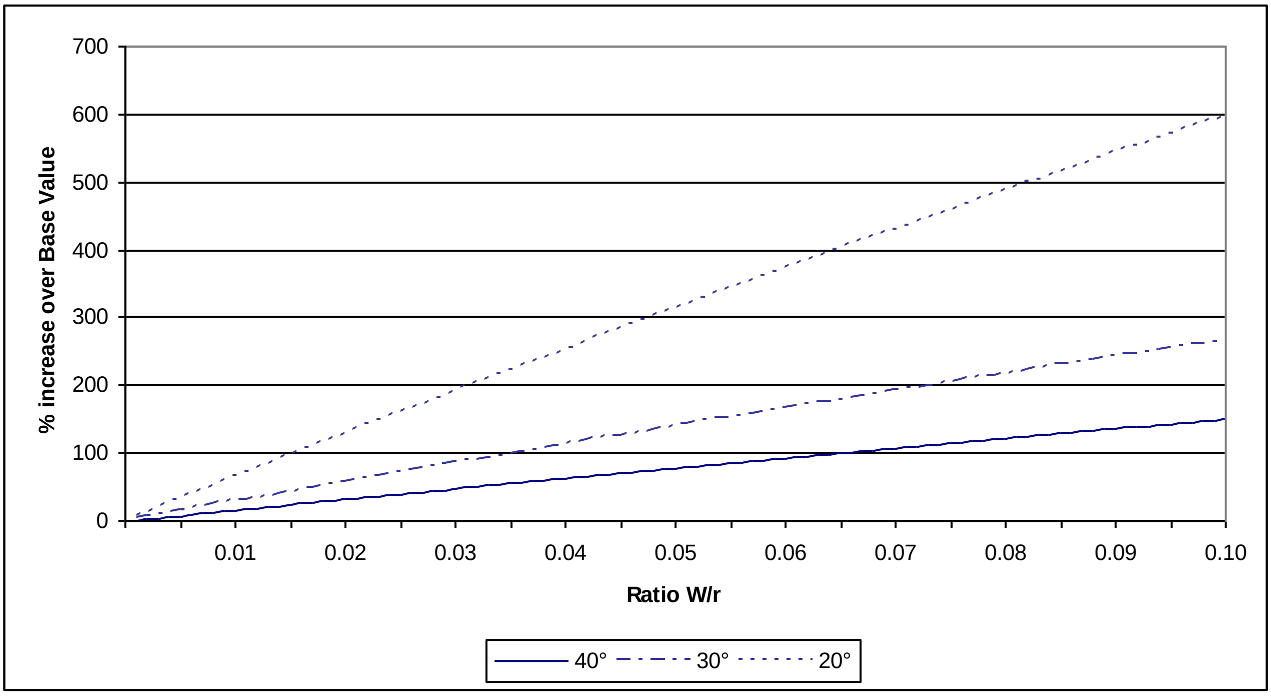
A useful measure to classify a bend can be generated by considering the ratio of *W* with *r*. Using this ratio bends which have the same ratio *W/r* the value of R_{max} as a percentage of the constant value (R_b) is found to be identical for all angles of turn. For example, if a bend of 100 m is considered, then if the available width is 1 m, the ratio *W/r* will be 0.01.

Of interest too is the percentage increase over the base constant value (R_b) with variations in angle of turn and available width. Graph 3 below shows how the percentage increase over the base constant value varies with the angle of turn. Graph 4 shows the linear relationship between the percentage increase over the base value and the available width.

Graph 3. To show variation in percentage increase of base value with angle of turn (Ratio *W/r* = 0.01, 0.02, 0.03)

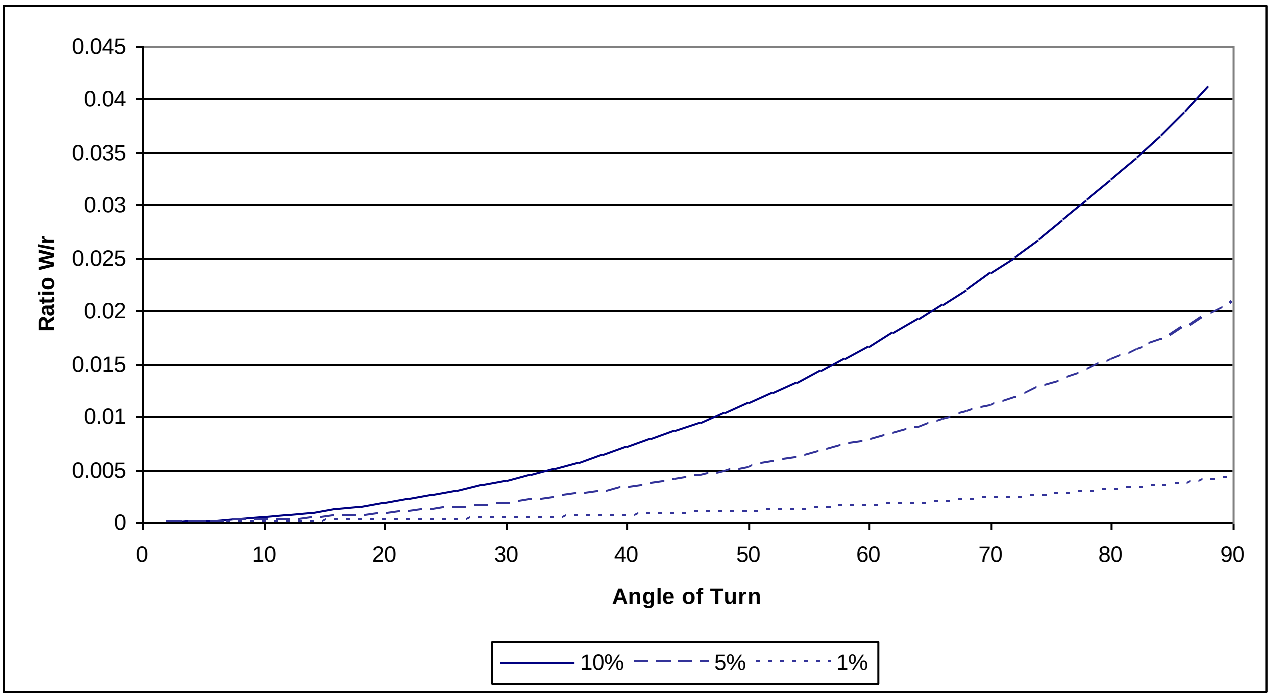


Graph 4. To show variation in percentage increase of base value with ratio *W/r* (Angle of turn = 20°, 30°, 40°)



As can be seen from the graphs, for small angles of turn the percentage increase over the base value is highly significant but between about 40° to 50° the percentage increase drops to relatively insignificant levels. An indication of ratio *W/r* and angles at which the percentage increase exceeds various levels of significance is shown in Graph 5.

Graph 5. To show variation in significance of ratio *W/r* with angle of turn (Significance levels = 1%, 5%, 10%)



If for example, it is decided that increases above 10% are significant, then for a 0.01 ratio bend, any angle less than about 48° will be significant as angles of turn smaller than 48° will result in increases greater than 10%.

Similarly, if a bend has an angle of turn of 50°, then increases above 5% of the base value can be expected when the ratio *W/r* exceeds 0.005.

To demonstrate how the R_{max} equation can be utilised in ordinary situations, the worked example in Appendix C may be of use.

References

1 Smith, R. *Critical Speed Motion*. 1991 IMPACT 2 (1) 12 – 14
2 Lambourn, RF. *The calculation of motor car speeds from curved tyre marks*. Journal of the Forensic Science Society 1989, 29: 371 - 386
3 Hague, DJ. Lambourn, RF. and Turner, DF. *The Accuracy of Speeds Calculated from Critical Curve Marks and their Striations*. 1997 Proceedings of 3rd National ITAI Conference
4 Neades, J. *Maximum Speeds for Bends*. 1991 IMPACT 2 (1) 15 - 16

Appendix A

Guidelines for the use of striated tyre marks to estimate vehicle speed

(After RF Lambourn[3])

1. There should be two marks visible from the outside wheels on the curve. Single marks may be used there should be clear other evidence that the rear wheels of the vehicle are tracking outside the path of the front wheel.

2. The measurement of the radius should be made from the front outside tyre mark using the chord and mid-ordinate method rather than from a scale plan. If possible aligning boards should be used to find the mid-ordinate length.

3. The measurement of the chord should begin at the earliest point compatible with the first condition. A length of 15m is generally suitable but a longer chord should be taken if the mid-ordinate is found to be less than 300 mm to minimise measuring errors. If the length of the marks dictates a shorter chord, the measurement should still proceed albeit with extreme care.

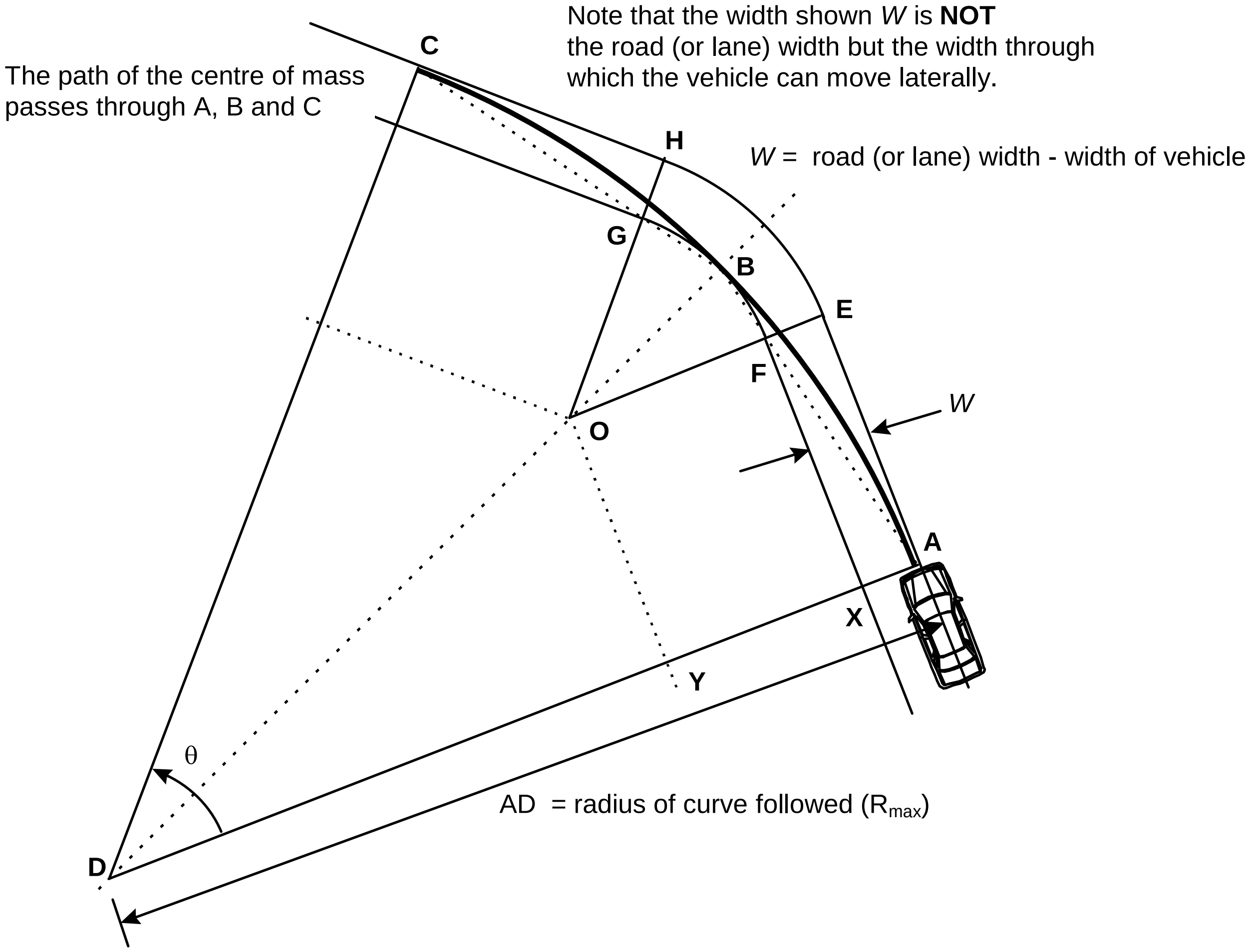
4. The separation of the front and rear tyre marks over the length of the chord should be no more than half the track width of the vehicle (although they may diverge further along the marks). Measurements made where there is a greater divergence are likely to give results which are an underestimate.

5. The gradient and crossfall of the road should be measured so that any necessary correction can be made.

Appendix B

Derivation of the R_{max} equation.

Path of vehicle around bend – Construction of vehicle path



The centre of mass of the vehicle follows the arc ABC which is part of a circle with the point D at the centre. The width shown, (W) is ***not*** the actual width of the road (or lane) but ***is*** the width available for the vehicle to move laterally when the width of the vehicle is taken into account. It is calculated as the width of the road (or lane) less the width of the vehicle.

W = Road (or lane) width - width of vehicle

The inside and outside radii constraining the lateral movement of the vehicle have a common centre at the point O in Figure 4. The angle through which the bend turns is shown as theta (θ).

Note that the path of the centre of mass of the vehicle starts the curve at A and finishes at C. This means that the outside lines, AE and HC are tangential to the arc ABC. The lines AFB and BGC are both straight lines as can be shown by considering triangles OFB, OBG, DAB and DBC which are all similar isosceles triangles.

As can be seen from Figure 4, the distance AD is the desired radius R_{max}. So that

$$R_{max} = AD$$

An expression for the desired radius AD may now be derived. The geometrical method used is to calculate the distance AD in the diagram by considering the individual distances AX, XY and YD

AX is the lateral width through which the vehicle can move (*W*) and is calculated using equation 3.

XY is the inside radius of the bend. This is not the actual measured radius of the bend itself since the centre of mass of the vehicle is unlikely to be able to reach this point. This is because the vehicle is not a point, but has a physical width. It is not unreasonable to assume that the centre of mass of a car is located approximately along the longitudinal centre line of the vehicle. Assuming that the centre of mass of the vehicle is located along this line, the inside radius of the bend XY is the inner radius of the bend itself plus half the width of the vehicle. In other words,

XY = r = inner radius of bend + ½ width of vehicle

The distance YD can also be determined.

It can be seen that,

$$OD = R_{max} - r$$

Since OD is the hypotenuse of triangle ODY the distance YD can be expressed as,

$$\mathrm{YD} = (R_{\max} - r)\ \cos\left(\theta/2\right)$$

As required, we have

$$R_{max} = AD = AX + XY + YD$$

So substituting values for AX, XY and YD we obtain,

$$R_{\max} = W + r + (R_{\max} - r)\cos\left(\theta/2\right)$$

This can be solved for R_{max} as follows,

$$R_{\max} = W + r + R_{\max}\cos\left(\theta/2\right) - r\cos\left(\theta/2\right)$$

$$R_{\max} - R_{\max}\cos\left(\theta/2\right) = W + r - r\cos\left(\theta/2\right)$$

$$R_{\max}\left(1 - \cos\left(\theta/2\right)\right) = W + r\left(1 - \cos\left(\theta/2\right)\right)$$

$$R_{\max} = r + \frac{W}{1 - \cos\left(\theta/2\right)}$$

Appendix C

Worked Example

A vehicle is alleged to have lost control on a left hand bend and collided head on with an oncoming vehicle. You need to establish the maximum speed at which a vehicle could negotiate the bend and remain on the correct side of the road. From the scene the following information is ascertained,

Width of lane	2.5 m
Width of vehicle	1.4 m
Inner radius of bend	80.0 m
Angle of turn	30°
Coefficient of friction	0.75

Firstly we need to find the values for *W* and *r* using equation 4 and 5 as these are not given directly.

$$W = 2.5 - 1.4 = 1.1 \text{ m}$$

$$r = 80.0 + ½ (1.4) = 80.7 \text{ m}$$

The ratio *W/r* for this bend is 1.1 / 80.7 or about 0.014. It can be seen immediately from Graph 5 that with an angle of turn of 30° and a ratio of 0.014 the increase in R_{max} will be somewhat larger than 10% and therefore significant.

We can use equation 3 to establish the maximum radius of turn,

$$R_{\max} = r + \frac{W}{1 - \cos\left(\theta/2\right)}$$

$$R_{\max} = 80.7 + \frac{1.1}{1 - \cos\left(30/2\right)}$$

$$R_{max} = 112.98 \text{ m}$$

As can be seen, the calculated value of R_{max} is larger than the base value of 81.8m by some 38%.

We can now substitute the known values in the critical speed equation,

$$V_{\max} = \sqrt{\mathrm{r}\,\mu\,\mathrm{g}}$$

$$V_{\max} = \sqrt{112.98 \times 0.75 \times 9.81}$$

$$V_{max} = 28.83 \text{ ms}^{-1}$$

Which is equivalent to approximately 64mph (103 kph).