

Incomplete Wheel Lockup

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Abstract

Techniques for establishing the speed of a vehicle from tyre marks left on the road surface are well known. Normally the assumption is made that all the wheels of the vehicle are contributing equally to the braking effort. In circumstances where it cannot be shown that all the wheels are contributing however, this assumption is invalid. This paper demonstrates methods for establishing an effective coefficient of friction for those situations where all the wheels are not contributing fully to the braking effort. A technique for establishing the height of the centre of mass is also presented. It also considers the implications of performing these calculations and considers the situation where a vehicle may not be able to develop the full effective coefficient due to excessive pitching and presents a method for establishing when excessive pitching may occur.

Notation

s	displacement
t	time
u	initial velocity
v	final velocity
a	acceleration
g	acceleration due to gravity
μ	coefficient of friction
m	mass
F	force
L	wheelbase
B	fraction of braking
R	normal reaction
h	height of centre of mass

Subscripts

F	front axle
B	back axle
FR	front right wheel
RL	front left wheel
BR	back right wheel
BL	back left wheel
e	effective value

Introduction

On many occasions vehicles skid over road surfaces leaving marks which can be measured by a reconstruction engineer. From the length of these marks the initial speed of the vehicle can be calculated provided a constant rate of acceleration is assumed. This gives the well known equation presented in many texts [1, 2],

$$u = \sqrt{v^2 + 2as} \quad (1)$$

For two surfaces in sliding motion relative to each other, a force will be generated which opposes that relative sliding motion. The magnitude of the frictional force so generated is given by the equation,

$$F = \mu R \quad (2)$$

Where F is the frictional force, μ is a coefficient of friction dependent upon the two surfaces and R is the normal reaction between the objects in contact. For an object which is sliding with a known coefficient of friction μ , the acceleration a in equation (1) may be replaced by μg to give,

$$u = \sqrt{v^2 + 2\mu g s} \quad (3)$$

Investigators generally determine the coefficient of friction by skidding a similar vehicle from a known speed.

In this form, equation (3) assumes that the vehicle is sliding across a surface with all wheels contributing to the braking effort. In situations where one or more of the load bearing wheels do not contribute to the braking effort, adjustments can be made to establish an *effective* coefficient of friction which can then be applied to the whole vehicle.

In this paper we consider first the situation where one or more of the wheels are not contributing at all to the braking effort and then to situation where they may contribute a partial amount. The method described here is similar to that described by Smith [3] but extends his results.

Derivation

Under braking the each wheel in contact with the ground contributes a proportion of the overall braking dependent on the actual weight carried by those wheels. The frictional force generated by each wheel is given by equation (2) where the normal reaction is the weight of the vehicle carried by that particular wheel, so the total braking force is given by the expression,

$$F = \sum \mu B_n R_n = \mu (B_{FL} R_{FL} + B_{FR} R_{FR} + B_{BL} R_{BL} + B_{BR} R_{BR}) \quad (4)$$

Where front and rear axles are considered together equation (4) can be simplified to give,

$$F = \sum \mu B_n R_n = \mu (B_F R_F + B_B R_B) \quad (5)$$

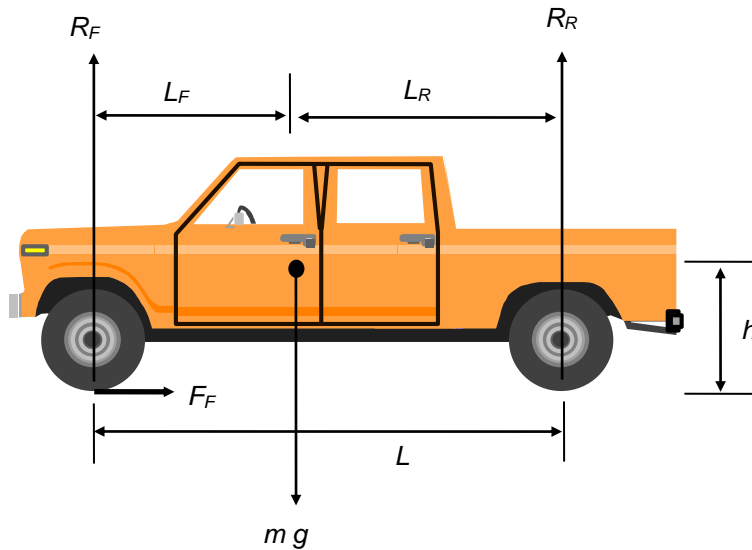
Where R_n is the reaction at each of the wheels or axles and B_n is the fraction of braking developed by that wheel or axle. B_n can range from 0.0 to 1.0 where 0.0 represents no braking and 1.0 represents full braking. For a vehicle where all the wheels are sliding, the sum of R_n is the entire weight of the vehicle, i.e. mg and B_n is 1.0 for each wheel. For a vehicle where all the wheels are sliding therefore, the overall frictional force is given directly by equation (2) where R is the normal reaction of the entire vehicle.

Where all the wheels are not braking however this situation is complicated by the fact that under braking, the actual load carried by each wheel change dynamically in response to a transfer of weight to the front of the vehicle. In simple terms there is a transfer of weight from the rear of the vehicle to the front due to the fact that the frictional force acts at the tyre / road interface which is below the centre of mass. This creates a moment arm about the centre of mass which causes a rotation. The amount of weight transferred under braking is dependent on the coefficient of friction itself, the wheelbase and the height of the centre of mass.

Allowances can be made for wheels which are not contributing to the braking effort by deriving an equation that generates an effective coefficient of friction. For simplicity, in this paper we consider the situation where only the front wheels are contributing to the braking effort. The same technique can be applied to other combinations and the full results are listed in Appendix A.

The derivation presented here is performed in two stages. First to establish the dynamic weight carried by the front wheels and second to establish the degree of braking generated by the braking wheels.

It is helpful to consider exactly which forces are acting on the vehicle together with the direction in which they act. The diagram below shows the forces acting on a skidding vehicle when only the front wheels are contributing to the braking effort so any frictional force is developed by the front wheels only.



As the vehicle travels along the road, it is not rotating about the centre of mass. The various moments must therefore be in equilibrium so it is possible therefore to equate the clockwise and anticlockwise moments.

In the first stage of this derivation, the dynamic weight carried by the front wheels is established. Equating the moments about the centre of mass, we obtain,

$$hF_F + L_R R_R = L_F R_F \quad (6)$$

Equation (2) can be substituted into the equation (6) to give,

$$h\mu R_F + L_R R_R = L_F R_F \quad (7)$$

We also know that the total weight of the vehicle is mg which allows us to define the reactions at the front and rear wheels, i.e.

$$mg = R_F + R_R \quad (8)$$

Substituting for R_R in equation (8) gives,

$$h\mu R_F + L_R (mg - R_F) = L_F R_F \quad (9)$$

The total wheelbase length is the sum of the two component parts, as follows,

$$L = L_F + L_R \quad (10)$$

Equation (10) can be substituted into equation (9) which itself can be transposed to give us an expression for R_F which is the solution to the first part of the derivation.

$$R_F = \frac{L_R mg}{L - \mu h} \quad (11)$$

The frictional force developed by the front wheels is given by equation (2). From Newton's Second Law we have $F = ma$ so that,

$$a_e = \mu_e g = \frac{F}{m} = \frac{\mu R_F}{m} = \frac{\mu L_R g}{L - \mu h} \quad (12)$$

This can be solved for the effective coefficient of friction, μ_e

$$\mu_e = \frac{\mu L_R}{L - \mu h} \quad (13)$$

Equation (13) although correct, present a slight problem in that it contains a term (L_R) describing the distance from the centre of mass to the rear axle. Using moments about the centre of mass it can also be shown that the static (non-braking) reactions at the front and rear axles together with the distances of the front and rear axles from the centre of mass are related by the equation,

$$R_{FS} L_F = R_{RS} L_R \quad (14)$$

Where R_{FS} and R_{RS} are the static fractions of the total weight at the front and rear axles respectively. Provided the centre of mass does not move within the vehicle, the distances L_F and L_R will be constant, whatever the motion of the vehicle, so that,

$$L_R = L R_{FS} \quad (15)$$

Substituting equation (15) into equation (13) gives the desired solution,

$$\mu_e = \frac{\mu R_{FS}}{1 - \mu h/L} \quad (16)$$

The reason for using the static weight fraction the front axle is that this is easily and directly measurable and thus relieves the investigator from having to calculate the position of the centre of mass. For example, if the total weight of the vehicle were 1000 kg and the weight on the front axle was 600 kg the fraction of weight on the front axle would be $600 / 1000 = 0.6$

Incomplete Four Wheel Braking

The technique described previously can be generalised to cater for the situation where a limited amount of braking is developed by each of the front and rear wheels. Consider now the situation where the rear wheels are contributing a fraction B_R of the maximum braking and the front wheels a fraction B_F . Equating the clockwise and anticlockwise moments as before now gives,

$$h\mu B_F R_F + L_R R_R + h\mu B_R R_R = L_F R_F \quad (17)$$

In this situation, braking is developed by both the front and the rear wheels, so the dynamic normal reaction under braking must be determined for each axle. Eliminating R_R and R_F in turn from equation (18) using equation (8) gives,

$$R_F = \frac{mg(L_R + \mu h B_R)}{L + \mu h(B_R - B_F)} \quad (18)$$

$$R_R = \frac{mg(L_F - \mu h B_F)}{L + \mu h(B_R - B_F)} \quad (19)$$

From Newton's Second Law the effective acceleration can be found and an expression for the effective coefficient of friction can be derived,

$$\mu_e = \frac{\mu(B_F L_R + B_R L_F)}{L + \mu h(B_R - B_F)} \quad (20)$$

Where the braking contribution from either the of the axles is zero, equation (20) reduces to the equations derived previously.

The fraction of braking developed by each wheel may not be known directly, but in many situations the braking efficiency developed by each wheel or axle can be determined.

Within the UK and elsewhere these figures are generated as part of the annual safety checks to determine whether brakes are operating at a satisfactory level. UK testing stations are all fitted with rolling road brake test equipment which measures the force generated by a particular wheel. This can be compared with the weight carried by the wheel, either by direct measurement as is the case with larger vehicles, or by using a table of approximate weights, which is the method for smaller vehicles.

Braking efficiency is often expressed as a percentage and may be found using the equation,

$$\text{Efficiency} = \frac{\text{Braking force}}{\text{Weight}} \times 100 \quad (21)$$

A braking efficiency of say 50% implies that that wheel or axle is capable of generating a braking force equal to 0.5 the load carried by that wheel or axle.

For the reconstruction engineer this information may in itself be useful. A tyre on a particular road surface may have a coefficient of friction of 0.7. If a braked wheel has an efficiency of 50% then the maximum braking which can be developed by the wheel is then 50% of the load, in other words the coefficient of friction would be 0.5. If the tyre / road coefficient was only 0.2, as might be the case on snow, a wheel with a braking efficiency of 50% could be expected to lock up and would only generate a braking force of 20% of the load with a coefficient of friction of 0.2.

In effect therefore the behaviour of the wheel can be modelled as if it had an effective coefficient of friction of either the braking efficiency or the tyre / road coefficient of friction, whichever is the lower value. In these cases equation (20) can be reformed into the expression,

$$\mu_e = \frac{\mu_F L_R + \mu_R L_F}{L + h(\mu_R - \mu_F)} \quad (22)$$

Height of Centre of Mass

The height of the centre of mass is a crucial parameter in all of the equations derived so far. Determining the height of the centre of mass is not straightforward and a practical solution is presented in Appendix B.

Reide [4] shows that the height of the centre of gravity lies between about 0.43m and 0.58m for most passenger cars. Research on motorcycles suggests that the height of the centre of gravity is at about saddle height although the data is sketchy to say the least [3]. With motorcycles the rider and pillion will have a significant effect on this calculation and must be included.

Several papers exist which list inertial data, including the height of the centre of gravity. These papers are written mainly for the North American collision investigation community although many of the vehicles are very similar to European and Australasian models. Of particular note is a rule of

thumb to determine the height of the centre of gravity presented by Garrott et al [5]. They found that the height of the centre of gravity for passenger cars was, 0.54 ± 0.04 m.

The height and longitudinal position of the centre of mass of a truck will be determined to a great extent by the nature of the load and the actual loading of the vehicle. Land Transport Safety Authority in New Zealand (LTSA) have developed a Static Roll Threshold calculator which requires as part of the input the height of the centre of mass and some advice is provided regarding the height of the centre of mass.

In general terms the height of the centre of mass can be calculated by considering the individual heights of the centres of mass of the component parts weighted by their respective masses. For vehicles it is usually sufficient to calculate the height of the sprung mass (that supported by the suspension) and the height of the unsprung mass (wheels and axles). In this case the overall height of the centre of mass is given by the equation,

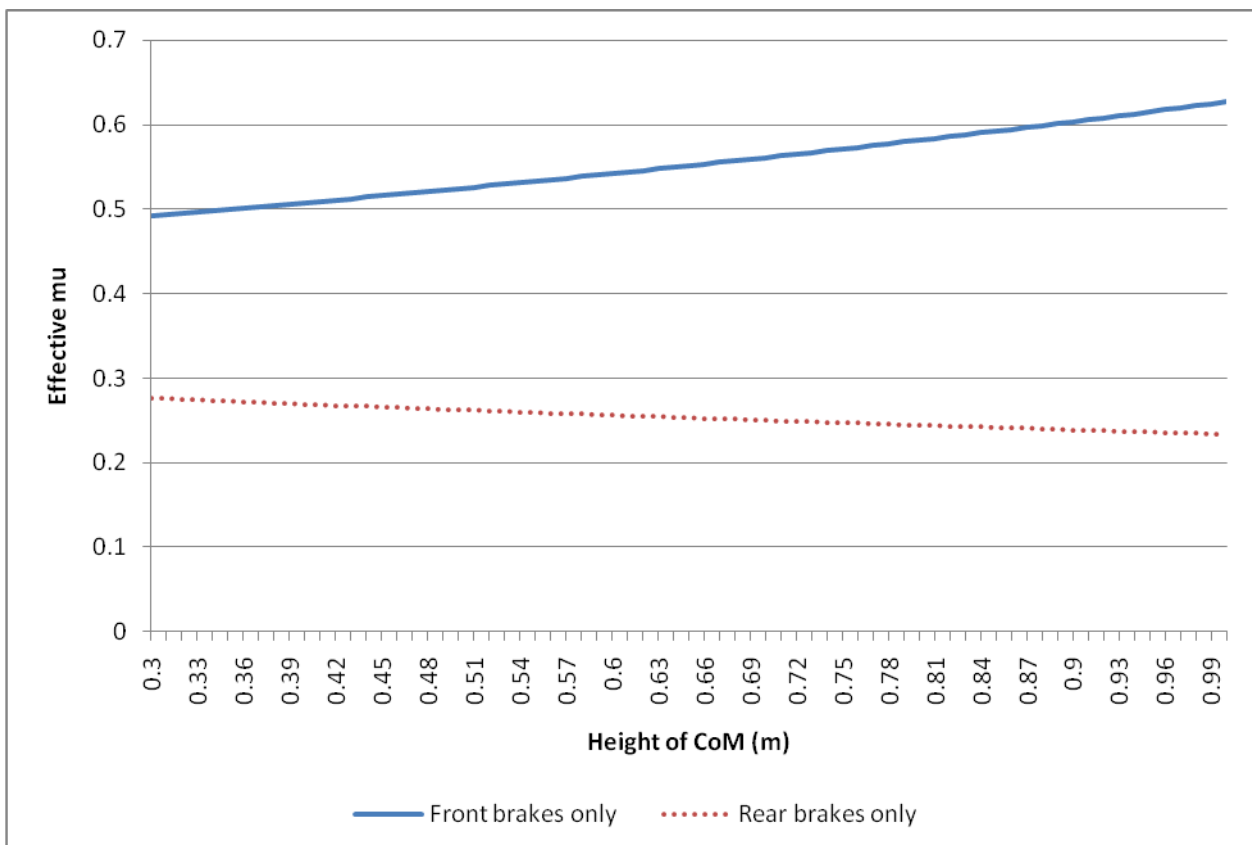
$$h = \frac{m_s h_s + m_u h_u}{m_s + m_u} \quad (23)$$

Where the subscripts *s* and *u* refer to the sprung and unsprung values respectively.

Discussion

As demonstrated it is possible to calculate the effective coefficient of friction. It is helpful also to consider the overall effect of the calculation for ranges of data. Studying the equations show that there are a number of variables which affect the result. Of prime consideration is the height of the centre of mass and the full value for the coefficient of friction. Graph 1 below shows the calculated effective coefficient of friction with varying heights for the centre of mass for a typical vehicle using a coefficient of friction of 0.75

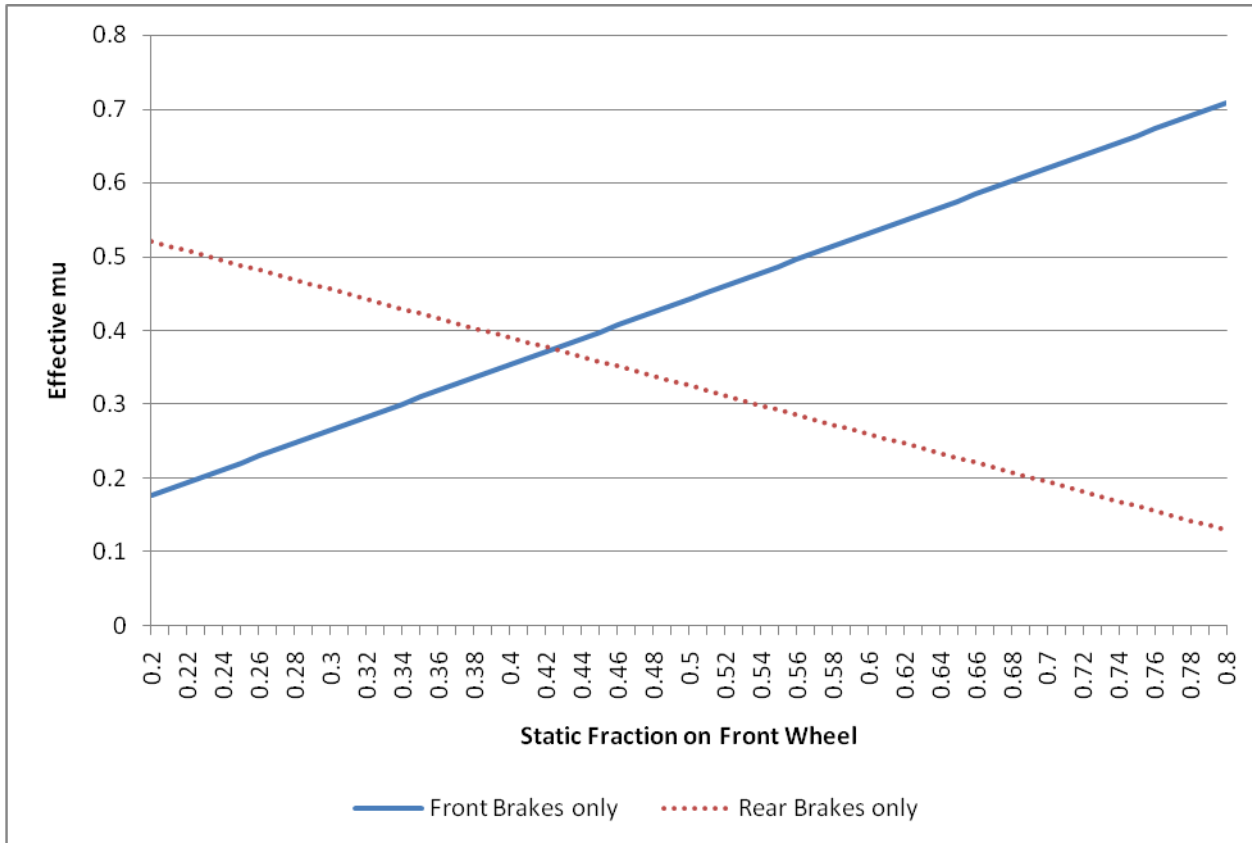
Graph 1: Effect of varying height of centre of mass ($\mu=0.75$)



For typical passenger cars using a typical coefficient of friction of 0.75, the range of effective coefficients determined for a range of CG heights of 0.43 to 0.58 produces results between 0.51 to 0.54 for front wheels and very little change for rear wheel braking of 0.27 to 0.26.

Graph 2 shows the effect of varying the longitudinal position of the centre of mass again using a typical passenger car. This changes the relative lengths of the moment arms L_F and L_R .

Graph 2: Effect of varying longitudinal position of centre of mass ($\mu=0.75$, $h=0.54m$)



As is to be expected, the graph shows that the effect of varying the relative lengths of the moment arms produces a linear change in the calculated coefficients. Typical passenger cars will generally have a static fraction of weight on the front wheel between 0.50 to 0.65. Using this range produces a range of values of 0.44 to 0.58 for front wheel braking only and 0.33 to 0.23 for rear wheel braking only.

It should be noted however that if the vehicle is heavily laden at the rear, this may reduce the proportion of weight carried by the front wheels to below 50%.

As can be seen from the graphs presented, there is not a great deal of variance in the results obtained within realistic ranges of data. This demonstrates that the calculations are not particularly sensitive to the input data. This suggests that for those situations where the exact data is unknown, a realistic result can be obtained by estimating the unknown variables. For example if the height of the centre of mass is unknown for a particular vehicle, then assuming a suitable value is likely to generate a useable result.

Extreme Pitching and Stoppies

It is interesting to examine the limits for which the derivation presented here holds true. The initial assumption was that under braking there is no rotation of the vehicle. For motorcycles and other short wheelbase vehicles, there may well be rotation about the front wheel under extremes of braking.

A technique practiced by some motorcyclists involves braking to such an extent that the rear wheel of the motorcycle leaves the ground. This is commonly known as a 'stoppie' or 'endo'. The technique involves applying sufficient braking force to raise the rear wheel and then balancing the bike on the front wheel while releasing the brake to some extent. Skilled practitioners can travel for hundreds of metres on the front wheel only.

The fact that stoppies can be performed, suggests that there is a limit to the amount of braking that can be developed by a motorcycle, whatever the actual coefficient of friction between the tyres and the road. At the point where the rear wheel leaves the ground, the normal reaction at the rear wheel will be zero. At that point too therefore all the weight of the vehicle must be carried by the front wheel. Equation (6) shows the equilibrium between the anticlockwise and clockwise moments. At the point where the rear wheel loses contact with the ground, R_R becomes zero and R_F becomes equal to mg , so in this situation equation (6) becomes,

$$hF_F = L_F R_F \quad (24)$$

Solving for F_F and substituting for R_F gives an expression for the maximum force that can be developed before the vehicle will start to rotate,

$$F_F = \frac{L_F mg}{h} \quad (25)$$

From Newton's Second Law we can determine the acceleration produced on the vehicle by dividing both sides by m . The resulting equation can then be divided by g to determine the maximum coefficient of friction which can be developed by the vehicle before it pitches about the front wheel,

$$\mu_{\max} = \frac{L_F}{h} \quad (26)$$

A typical wheelbase for a modern motorcycle is approximately 120 - 140 cm. Modern motorcycles are generally designed such that when ridden the centre of mass is positioned approximately midway between the wheels and at a height at or just under the saddle level. For a typical motorcycle therefore this will give a 50:50 front to rear weight distribution and a height for the centre of mass of about 60 - 80 cm. Using equation (26) suggests therefore that the maximum effective coefficient of friction will be in the range 0.85 to 1.0.

Tyres on motorcycles are able to generate peak tyre/road coefficients of friction well in excess of 1.0 as shown by Lambourn and Wesley [6], so the limiting factor imposed by equation (26) is not merely academic. Note that this analysis does not include any effect caused by the forks compressing under braking, which will tend to lower the height of the centre of mass and does not include any other effects. For example motorcyclists often move their position on the saddle further backwards under heavy braking. In turn this moves the centre of mass rearwards.

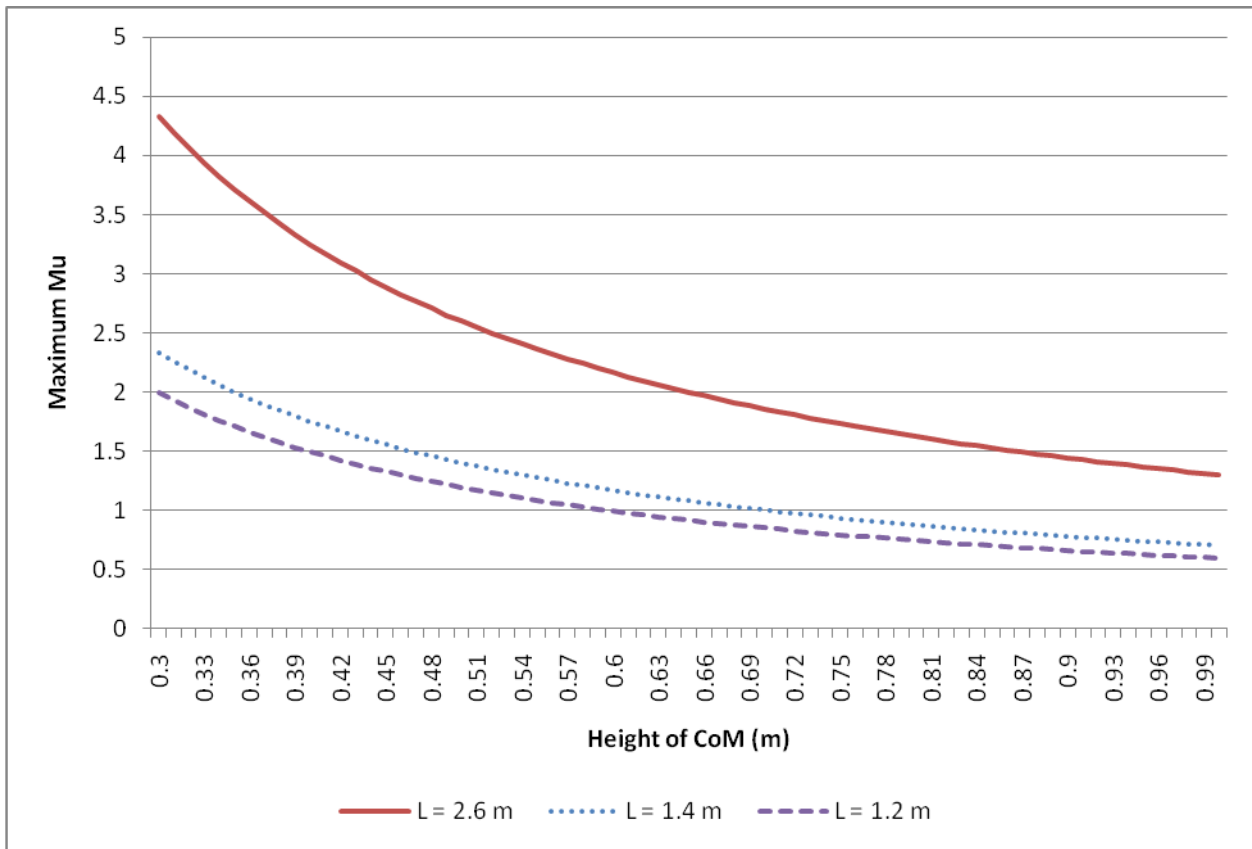
The effect of a maximum limit to the value of the coefficient that may be generated by a particular vehicle can be illustrated by considering vehicles of varying wheelbase lengths. Graph 3 shows the calculated maximum value for the coefficient of friction for various heights of the centre of mass for three vehicle wheelbases.

The solid line represents a passenger car with a wheelbase of 2.6 m. As can be seen the maximum coefficient at the expected heights for the centre of mass around 0.4 m is in the region of 2.5. This is well in excess of a realistic coefficient for car tyres on a road surface.

Motorcycles however have a much shorter wheelbase. The dotted and dashed lines in Graph 3 show the variation in the maximum coefficient of friction for motorcycles with a wheelbase of 1.2 and 1.4 m. With these vehicles a higher centre of mass shows that the coefficient of friction is very

much reduced. Indeed with a sufficiently high centre of mass, the coefficient of friction drops down below the level expected for car tyres on road surfaces.

Graph 3: Calculated maximum μ for various wheelbase lengths (50% weight distribution)



References

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- 6 Lambourn, RF. and Wesley, A. Motorcycle Tyre Road Friction, 2010 SAE 2010-01-0054

Appendix A

Incomplete wheel lockup equations

Two and four wheel vehicles

Both front wheels locked only Both rear wheels locked only

$$\mu_e = \frac{\mu R_{FS}}{1 - \mu (h/L)}$$

$$\mu_e = \frac{\mu R_{RS}}{1 + \mu (h/L)}$$

Where:

μ = Full μ

μ_e = Effective μ

h = height of centre of mass

R_{FS} = Static fraction of weight on *front* wheels

R_{RS} = Static fraction of weight on *rear* wheels

B_R = fraction of braking on rear wheels

B_F = fraction of braking on front wheels

L = Wheelbase

Four wheel vehicles only

One front wheel locked only One rear wheel locked only

$$\mu_e = \frac{\frac{1}{2}\mu R_{FS}}{1 - \frac{1}{2}\mu (h/L)}$$

$$\mu_e = \frac{\frac{1}{2}\mu R_{RS}}{1 + \frac{1}{2}\mu (h/L)}$$

Both front and one rear Both rear and one front

$$\mu_e = \frac{\mu(1 - \frac{1}{2} R_{RS})}{1 - \frac{1}{2}\mu (h/L)}$$

$$\mu_e = \frac{\mu(1 - \frac{1}{2} R_{FS})}{1 + \frac{1}{2}\mu (h/L)}$$

One rear, one front wheel locked only

$$\mu_e = \frac{1}{2}\mu$$

Partial braking from front and rear axles

$$\mu_e = \frac{\mu(B_F L_R + B_R L_F)}{L + \mu h(B_R - B_F)}$$

$$\mu_e = \frac{\mu_F L_R + \mu_R L_F}{L + h(\mu_R - \mu_F)}$$

Calculation of height of centre of mass

$$h = \frac{L(R_{F1} - R_F)}{mg \tan \theta} + r$$

Where: $R_{F1} - R_F$ = weight transfer

r = rolling radius

θ = angle raised

$$h = \frac{\text{wheelbase} \times \text{weight transfer}}{\text{total weight} \times \tan \theta} + \text{rolling radius}$$

Appendix B

Finding the Height of the Centre of Gravity

Before explaining the theory behind this method it is helpful to discuss the measurements that need to be obtained before the calculations can be performed. These are listed below,

1. Wheelbase of vehicle (L)
2. Total weight of the vehicle (mg)
3. Weight on the front axle whilst vehicle level (R_F)
4. Height through which vehicle rear wheels raised (T)
5. Weight on front axle when rear wheels are raised (R_{F1})
6. Rolling radius of the wheels (r)

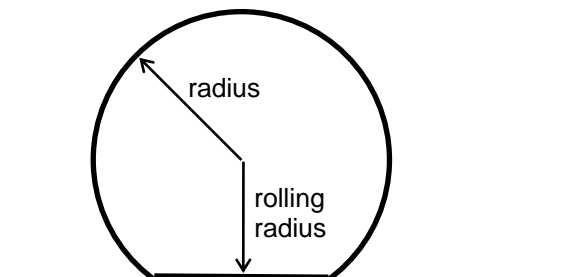
The wheelbase of the vehicle can easily be obtained by direct measurement or from manufacturers data. The total weight of the vehicle can be obtained by putting the vehicle onto a weighbridge as too can the weight on the front axle alone.

The next few measurements are more awkward to obtain. The rear wheels must be raised through a known height and the weight transferred to the front axle measured.

It may be supposed that it would be a simple matter just to jack up the rear of the vehicle. Unfortunately rear wheels too need to be raised through a known height. Jacking up the rear of the vehicle does not allow for any suspension movement. The most practical method is to place the rear wheels on a block of known height and then measure the weight on the front wheels. Alternatively, placing the wheels on the back of a sloping truck bed is a practical solution.

As we shall see in the theory behind this method, we need to raise the rear wheels through a reasonable distance to reduce the significance of errors in the measurements. We suggest that for an ordinary car or motorcycle, you should aim for a height of about 0.5 m.

The final measurement, that of the rolling radius, is also interesting in its own right. Consider a tyre resting on the ground as shown in the diagram below,



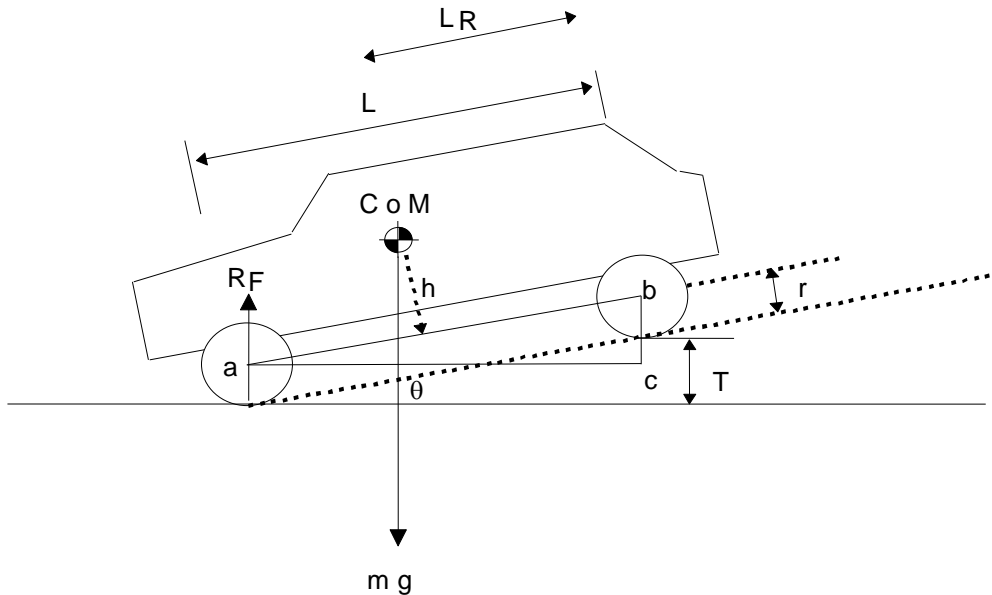
The base of a tyre deforms when in contact with the ground as shown in the diagram. This has the effect of reducing the radius of the tyre. We could measure this rolling radius directly, but it is difficult to perform this measurement accurately. Another method would be to roll or drive the vehicle forward through an exact number of revolutions of the wheel and measure the distance travelled. The rolling radius could then be calculated using the equation,

$$\text{Rolling radius} = \frac{\text{Distance}}{\text{No. of turns} \times 2\pi}$$

If you choose to adopt this latter method of determining the rolling radius, then we suggest that a minimum of five revolutions is needed to obtain reasonable results.

(Tyre pressures too can affect the results slightly, so it may be prudent to find the rolling radius separately for each of the front wheels and then average the results.)

Now that the six measurements have been obtained, we can discuss how we can use them to determine the height of the centre of gravity. The basic configuration is shown in the diagram below,



We have superimposed a triangle abc on the diagram. The distance ab in the triangle represents the wheelbase L and bc represents the height through which the rear of the vehicle has been raised T as discussed earlier.

We need to know the angle of the slope of the vehicle. Using trigonometry the sine of the angle theta can be calculated.

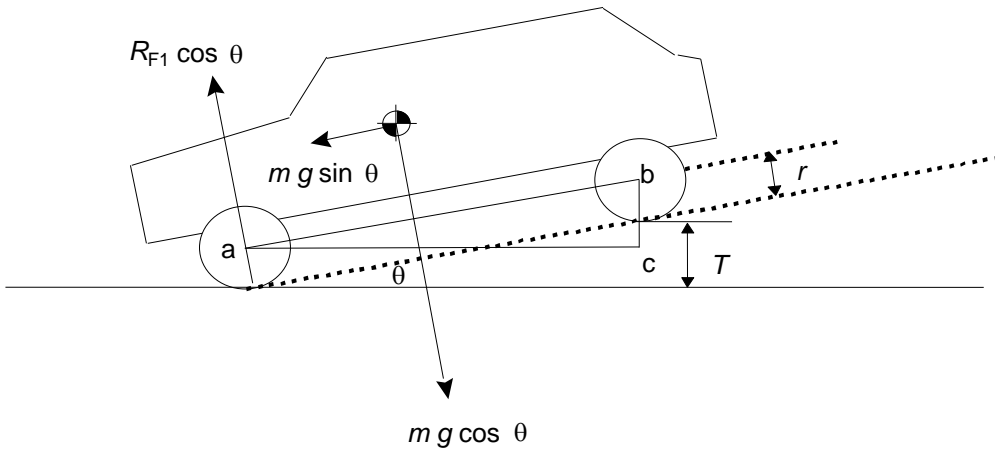
$$\sin \theta = \frac{T}{L} = \frac{bc}{ab}$$

The basic method we use to calculate the height of the centre of gravity is based on the techniques discussed in the text by taking moments about a point. The vehicle is not rotating so the clockwise and anticlockwise moments are equal.

In our discussion we have assumed that the front wheel remains on the ground and the rear wheel is raised. There is no particular reason for the technique to be performed this way round as we could just as easily raised the front wheel through a known height.

With the vehicle raised as shown in the diagram, we shall take moments around the rear axle to establish the position of the centre of gravity. Before we can do this, we need to resolve the two force vectors indicated by mg and R_F .

Remember that in order to calculate moments we need the perpendicular distance at which the force is acting, hence the reason for resolving the vectors so they are either parallel or perpendicular to the slope. The diagram below indicates these vectors. (Note that the mg and R_F vectors have been omitted from this diagram for clarity.)



Since the vehicle is in equilibrium, we can take moments about the rear axle, (point b), to give,

$$R_{F1}L \cos \theta = mgL_R \cos \theta + mgh \sin \theta$$

Dividing throughout by $\cos \theta$ gives,

$$R_{F1}L = mgL_R + mgh \tan \theta$$

This can be rearranged for h to give,

$$h = \frac{R_{F1}L - mgL_R}{mg \tan \theta} \quad (A)$$

It is helpful to eliminate L_R from the equation. This can be achieved since it is already known that,

$$L_R = L \frac{R_F}{mg}$$

We can substitute this expression into equation (A) to give,

$$h = \frac{L(R_{F1} - R_F)}{mg \tan \theta} \quad (B)$$

Where the expression $(R_{F1} - R_F)$ in this equation is the difference between the weight on the front axle when raised, R_{F1} and the weight on the same axle when level R_F . Consider however the diagram above. This shows us that equation (B) gives us an expression for the height of the centre of mass above the axle line (ab). To this value we need to add to this figure the rolling radius r to give us the actual height of the centre of mass above the ground, so that,

$$h = \frac{L(R_{F1} - R_F)}{mg \tan \theta} + r \quad (C)$$

Or in words,

$$h = \frac{\text{wheelbase} \times \text{weight transfer}}{\text{total weight} \times \tan \theta} + \text{rolling radius}$$