How Close is Too Close?

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AiTS, Aerial View, Shab Hill, Birdlip, Glos.

Abstract
It is sometimes helpful to establish whether the distance between following vehicles is sufficient to allow the following vehicle to stop safely without striking the lead vehicle should the lead vehicle brake suddenly. The method described here provides an expression for calculating the minimum following distance and time. This technique is based on a relationship between the reaction time of the following driver, the rate of acceleration and the initial gap between the vehicles. Also presented is a method for establishing whether a collision will occur between following vehicles and if so, when that collision will occur.

NOTATION

t	time
t_r	reaction time
s	distance
u	initial speed of the vehicle
v	final speed of the vehicle
a	acceleration
G	initial gap between the vehicles

Subscripts

1	vehicle 1
2	vehicle 2
b	braking
r	reaction
t	total

DERIVATION

Definition of the problem

In some situations it is necessary to determine whether a vehicle is following another vehicle too closely. In many countries it is a specific offence to follow another vehicle too closely, or if not a specific offence (such as in the UK), more general offences might be considered such as dangerous driving.

Generally, if vehicles are travelling in convoy, then it is reasonable to assume that the two vehicles are travelling at the same speed and this assumption is made throughout this paper.

The first problem when dealing with such situations is to determine exactly what is meant by the phrase, 'following too closely'. Two solutions are possible, one based on the premise that a driver has to react and stop completely within the available gap and the second where the driver merely has to react. The first method is inherently safer as even if the lead vehicle comes to a instant stop, for example by colliding with another object, the following vehicle can still stop.

However this option may not be too realistic, especially with increasing congestion on the roads. Many drivers drive their vehicles with a very much reduced gap. It can be shown that if the driver can react within the following distance, then if the lead vehicle brakes to a stop the following driver can also brake to a stop without a collision.

React and Stop Solution

If the first solution is adopted, then the safe following distance can be calculated from the distance required in which the driver can react and then bring their vehicle to a stop. Written as equations, the two phases are as follows,

$$s_r = ut_r \quad (1)$$

$$s_b = \frac{u^2}{2a} \quad (2)$$

The total distance in which to react and stop (*S*) is the sum of equations (1) and (2) leading to,

$$s_t = ut_r + \frac{u^2}{2a} \quad (3)$$

This is a quadratic in *u* which can be solved for *u* to give,

$$u_{\max} = a\left[-t_r \pm \sqrt{t_r^2 + \frac{2s_t}{a}}\right] \quad (4)$$

As with all quadratics, there are two possible solutions, hence the ± symbol in equation (4). In practice the solution using the plus sign is the one which provides the necessary solution. This particular equation is taught in collision investigation courses[1] as the equation to use where limited visibility problems are considered, but it applies equally to vehicles following one another.

It is often relevant to relate the safe following distance to a time gap between the two vehicles. Indeed the UK Government once ran a road safety campaign using the slogan "*Only a fool breaks the two second rule"* The idea being to encourage drivers to adopt a two second time gap between themselves and the vehicle in front.

The corresponding time behind which the second vehicle should follow, is calculated as follows,

$$t = \frac{s}{u} \quad (5)$$

Allowing for a 1.0 – 2.0 second reaction time, and a typical road surface affording a good braking surface (a = 6.86 ms^{-2}) suggests the following distances and times.

Table One: Following distances – React and Stop

Speed (km/h)	Speed (ms^{-1})	Min Distance (m)	Min Time (s)	Max Distance (m)	Max Time (s)
50	13.89	27.95	2.01	41.84	3.01
60	16.67	36.91	2.21	53.58	3.21
70	19.44	47.00	2.42	66.45	3.42
80	22.22	58.22	2.62	80.44	3.62
90	25.00	70.55	2.82	95.55	3.82
100	27.78	84.02	3.02	111.79	4.02
110	30.56	98.61	3.23	129.16	4.23
120	33.33	114.32	3.43	147.65	4.43

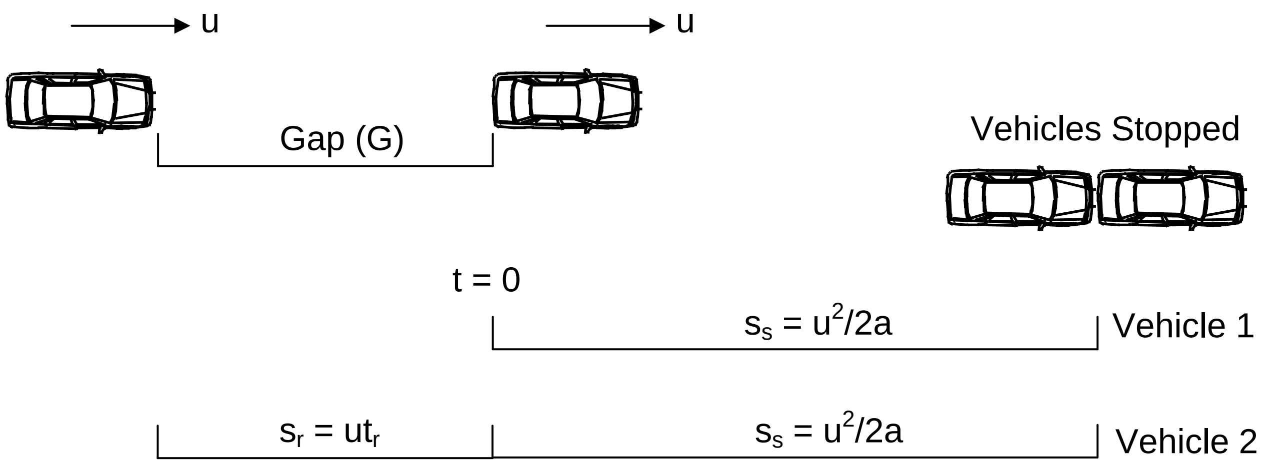
React Only Solution

Equation (4) and the safe following distances listed in Table One, may not always be a reasonable solution to the following vehicles problem. It can be argued that it is unlikely that the lead vehicle would suddenly stop dead, thus requiring the following driver to react and stop in the gap between the two vehicles.

It may therefore be more realistic to consider the situation where the lead vehicle brakes suddenly and brings the lead vehicle to a halt under emergency braking. In such a situation, a following driver would need to react and stop before colliding with the rear of the lead vehicle. The method described below shows how this situation can be modelled.

If the same rate of braking is assumed for each vehicle, then obviously both vehicles will stop in the same distance from the same speed. Since it is assumed that vehicles which are travelling in convoy are travelling at the same speed, then it follows that both vehicles will stop in the same distance. In general, this situation can be described as shown in Figure 1 below,

Figure 1.



In this situation, provided the following driver reacts and starts braking at or before the point where the lead vehicle starts braking, then a collision will be avoided. This suggests that the minimum following distance (*G*) in this situation is the reaction or thinking distance,

$$G = u\,t_r \tag{6}$$

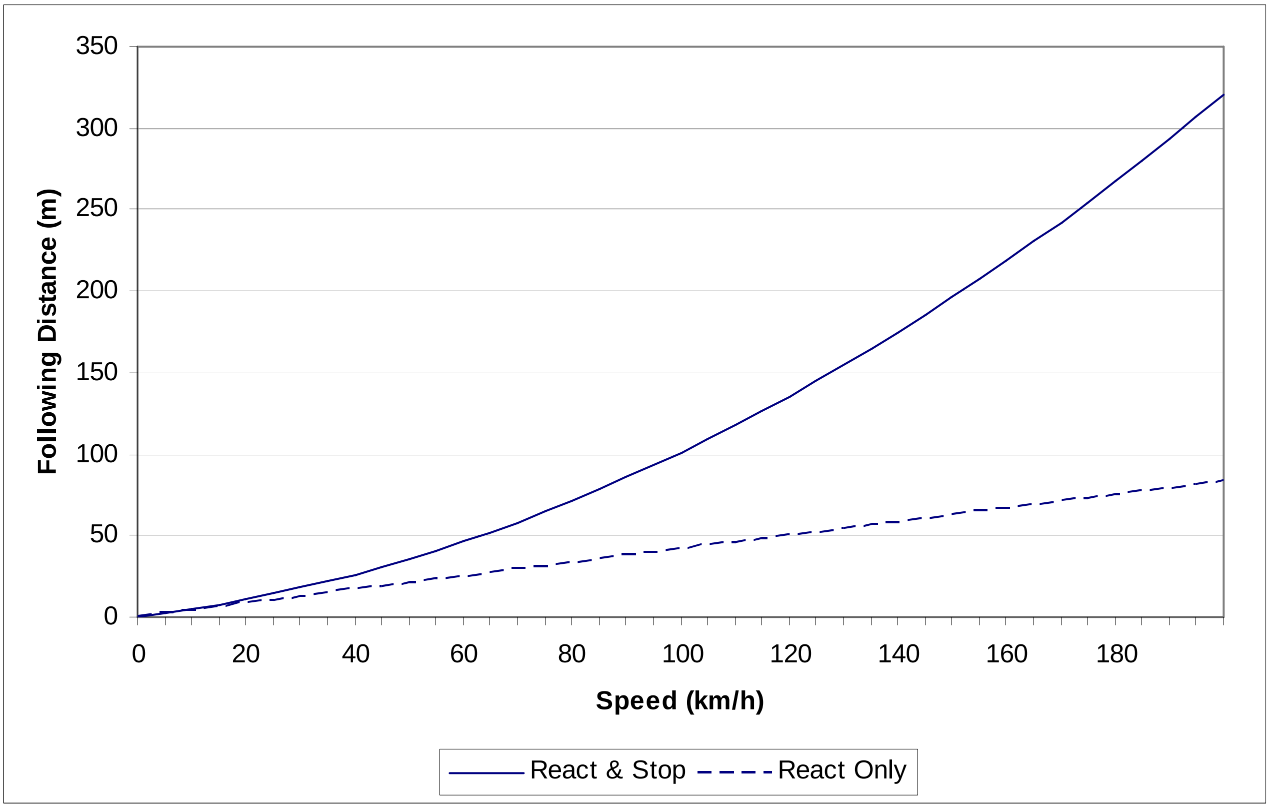
A similar table of following distances can be compiled for this solution using the same data as in Table One. This is shown in table Two,

Table Two: Following distances – Reaction only

Speed (km/h)	Speed (ms^{-1})	Min Distance (m)	Min Time (s)	Max Distance (m)	Max Time (s)
50	13.89	13.89	1.00	27.78	2.00
60	16.67	16.67	1.00	33.34	2.00
70	19.44	19.44	1.00	38.88	2.00
80	22.22	22.22	1.00	44.44	2.00
90	25.00	25.00	1.00	50.00	2.00
100	27.78	27.78	1.00	55.56	2.00
110	30.56	30.56	1.00	61.12	2.00
120	33.33	33.33	1.00	66.66	2.00

A comparison between the two methods is helpful. The graph below shows a comparison between the react and stop and the react only solutions.

Graph 1. Comparison between React and Stop and React Only Solutions



The second solution where a following driver reacts only, is of course linearly related to the speed of the vehicles.

Of more interest is in determining whether a collision will occur between the vehicles and if so, when that collision would take place. Another consideration will be the relative speed of the vehicles at the moment of impact, since this will determine the severity of the impact. These considerations form the basis for the discussion that follows.

Determining When a Collision Occurs

We consider first the situation where the following distance or gap (*G*) is known and derive an equation which provides the time of collision after the first vehicle starts braking.

The base equation required is one of the standard equations of motion,

$$s = ut + \tfrac{1}{2}at^2$$

Acceleration is negative for a slowing vehicle, so for the lead vehicle (vehicle 1) this gives

$$s_1 = ut - \tfrac{1}{2}at^2 \qquad (7)$$

Vehicle 2 is a little more complicated. The distance that vehicle 2 travels is determined by the linear function *ut* before it starts to brake and by the same quadratic equation as vehicle 1 once it too is braking. Equation 8 describes the position of vehicle 2 before it starts braking and equation 9 once vehicle 2 starts braking.

$$s_r = ut - G \quad (8)$$

$$s_2 = u(t-t_r) - \tfrac{1}{2}a(t-t_r)^2 + ut_r - G \quad (9)$$

Where t_r is the reaction time and G is the initial gap between the two cars. Since vehicle 2 is following vehicle 1 and is a known distance behind vehicle 1 we need to subtract the gap (G) in both these equations so that the overall displacement matches that for vehicle 1. The expression ($t - t_r$) in equation 9 is also required so that the time can be expressed so that t=0 is defined as the moment vehicle 1 starts to brake.

The latter two terms in equation 9 are also required to account for the displacement of vehicle 2 at the moment it starts to brake, For example an initial speed of 20 m/s and a one second reaction time, vehicle 2 will travel 20 m in one second. If we assume a 4 m gap behind the other, this means that when vehicle 2 starts to brake, it will have travelled 16 m beyond the point that the first car started skidding.

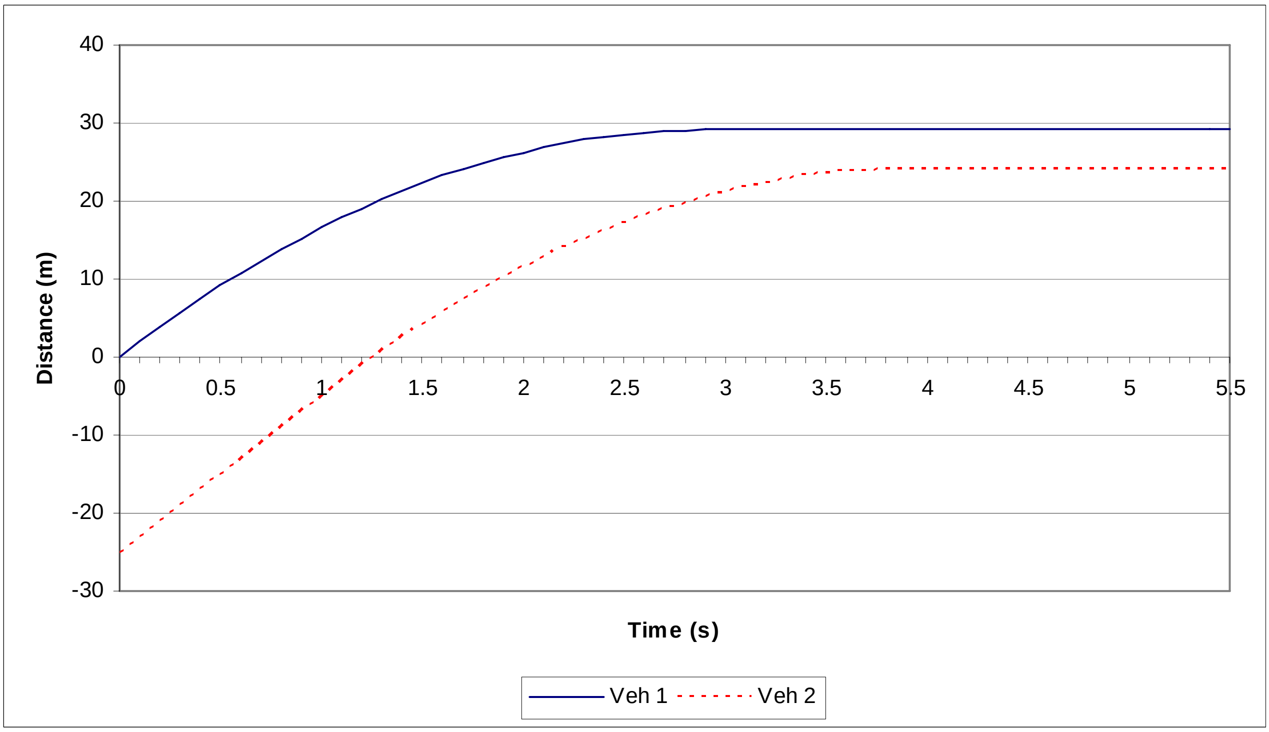
At impact, the displacement of each vehicle must be the same, ie s_1=s_2

Three possibilities are evident depending on the initial gap, reaction time of the second driver and acceleration. If the gap is sufficiently large (or the reaction time sufficiently short) vehicle 2 may stop before it collides with the rear of vehicle 1.

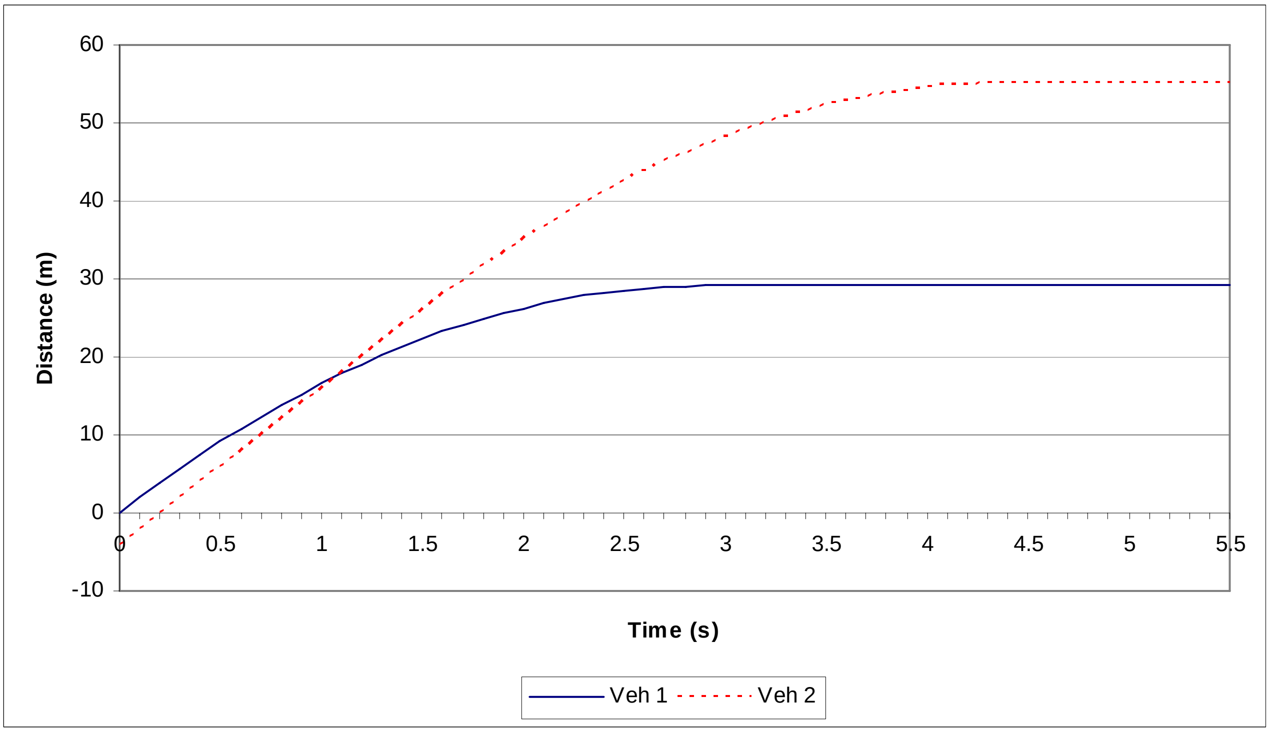
Alternatively, an impact may occur before vehicle 2 starts to brake, or finally it may occur while vehicle 2 is braking.

These three possibilities can be shown in graphical format using distance time graphs as shown in Graphs 2, 3 and 4.

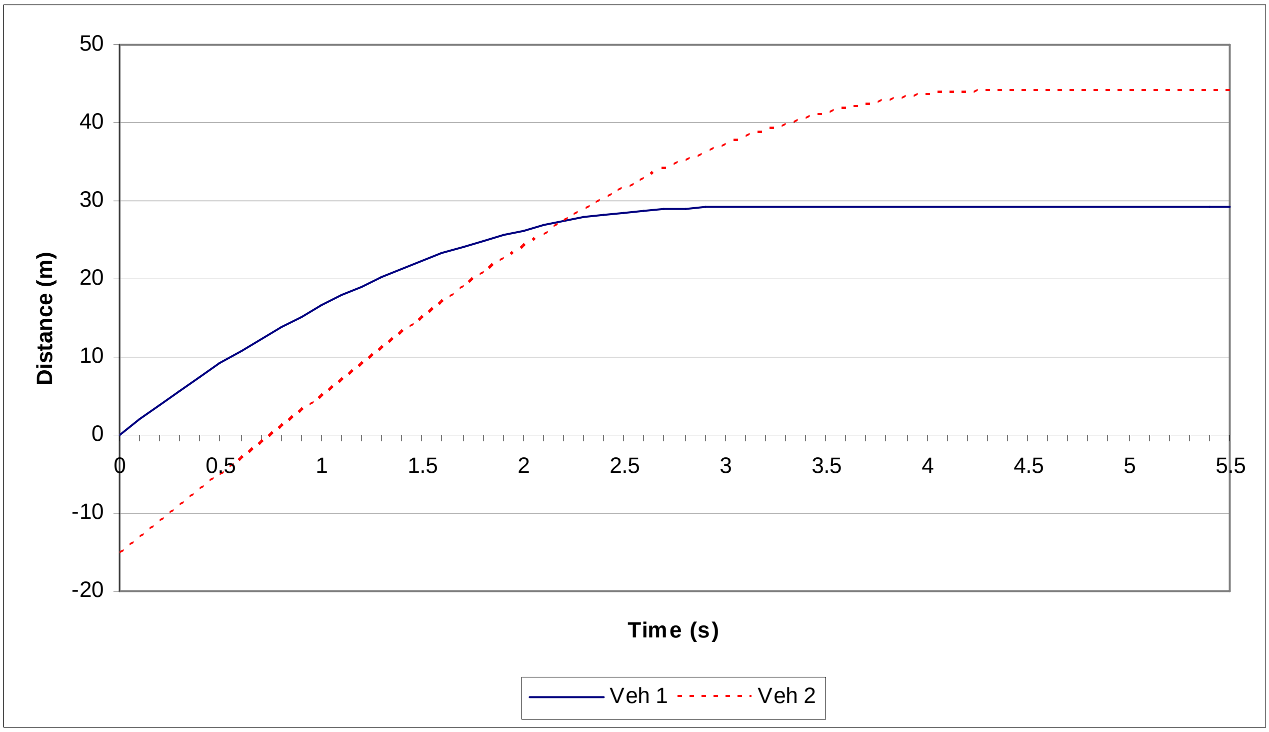
Graph 2. Collision does not occur



Graph 3. Collision occurs before vehicle 2 starts braking



Graph 3. Collision occurs after vehicle 2 starts braking



The three options suggest three possible solutions all of which must be considered in order to determine whether or not a collision occurs and when.

For a collision not to occur, the driver of vehicle 2 must start braking at or before the point at which vehicle 1 started to brake. As outlined above this occurs when,

$$G \geq ut_r \qquad (10)$$

Equation 10 provides a solution to the problem of determining whether or not a collision actually occurs. If the solution to equation 10 however shows that the gap is less than ut_r then a collision is inevitable and the next problem is to determine exactly then a collision occurs.

If it is assumed that the collision occurs during the linear phase, in other words before vehicle 2 has started to brake, then the motion of the vehicles is described by equation 7 for vehicle 1 and equation 8 for vehicle 2. For an impact to occur the displacement must be the same $s_1=s_2$. Equating equations 7 and 8 gives,

$$ut - \tfrac{1}{2}at^2 = ut - G$$

This can be solved for t to give,

$$t = \sqrt{\frac{2G}{a}} \qquad (11)$$

For equation10 to be valid, the driver of vehicle 2 cannot have begun braking. It follows therefore that if the reaction time of driver 2 is greater or equal to the solution of this equation, then a collision will occur at the time given by this equation.

If however the reaction time of driver 2 is less than this critical value, then vehicle 2 must have started braking so the motion of vehicle 2 is then described by equation 9. As before, if there is a collision then the displacements of each vehicle will be the same, so equating 7 and 9 gives,

$$ut - \tfrac{1}{2}at^2 = u(t - t_r) - \tfrac{1}{2}a(t - t_r)^2 + ut_r - G$$

This can be solved for t as follows,

$$ut - \tfrac{1}{2}at^2 = u(t - t_r) - \tfrac{1}{2}a(t^2 - 2tt_r + t_r^2) + ut_r - G$$

$$ut - \tfrac{1}{2}at^2 = ut - ut_r - \tfrac{1}{2}at^2 + att_r - \tfrac{1}{2}at_r^2 + ut_r - G$$

Cancelling where possible, this produces,

$$0 = att_r - \tfrac{1}{2}at_r^2 - G$$

Finally solving for *t* produces,

$$att_r = \tfrac{1}{2}at_r^2 + G$$

$$t = \frac{\tfrac{1}{2}at_r^2 + G}{at_r} \qquad (12)$$

In order to provide a coherent algorithm for solving problems concerning following distances, it is suggested that equation 10 is calculated first to determine whether or not a collision actually occurs. The second stage is to evaluate equation 11 to determine the maximum time for the collision to take place if it occurs during the linear phase.

If the solution to equation 11 suggests that a collision occurs during the braking phase of vehicle 2, then equation 12 should be evaluated to determine the time of impact.

WORKED EXAMPLE

In order to demonstrate the utility of this method, the following worked example may be of use. For this example it is assumed that both vehicles are travelling at 20 ms^{-1} and that both are capable of braking at a rate of 6.87 ms^{-2}. It is further assumed that the reaction time of the second driver is 1.0 seconds and that the vehicles are separated by an initial gap of 4 metres.

A spreadsheet solution to this problem is shown in Appendix A.

The first stage is to establish whether a collision occurs using equation 10. The product ut_r for this particular problem is 20 m which is considerably more than the initial gap of 4 m, thus a collision is inevitable.

Equation 11 provides the time of impact, if the impact were to occur during the linear phase of the motion of vehicle 2. Substituting given values gives,

$$t = \sqrt{\frac{2G}{a}} = \sqrt{\frac{2 \times 4}{6.87}} = 1.079\,\mathrm{sec}$$

1.079 seconds is marginally larger than the given reaction time, so the collision will actually occur at the time given by equation 12. Substituting known values into equation 12 gives,

$$t = \frac{\frac{1}{2}at_r^2 + G}{at_r} = \frac{\frac{1}{2} \times 6.87 \times 1.0^2 + 4}{6.87 \times 1.0} = 1.082\,\mathrm{sec}$$

Thus there will be a collision between the two vehicles and this will occur at 1.082 seconds after vehicle 1 starts to brake.

DISCUSSION

Once the time of impact has been determined, then it is straightforward to establish the displacement of the vehicles at impact and their respective speeds. Using the data from the Worked Example above, at 1.082 seconds vehicle 1 has travelled a distance given by the equation,

$$s = ut - \tfrac{1}{2}at^2 = 20 \times 1.082 - \tfrac{1}{2} \times 6.87 \times 1.082^2 = 17.62\ \mathrm{ms^{-1}}$$

Also the speed of vehicle 1 at the moment of impact is given by,

$$v = u - at = 20 - 6.87 \times 1.082 = 12.56\ \mathrm{ms^{-1}}$$

The speed of vehicle 2 at impact is given by,

$$v = u - a(t - t_r) = 20 - 6.87 \times (1.082 - 1.0) = 19.44\ \mathrm{ms^{-1}}$$

Of note it the relative speeds of the two vehicles at impact. In this example it is 6.87 ms^{-1}. It can be demonstrated that provided the collision occurs during the braking phase of each vehicle, then the relative speed at impact of the two vehicles is always the same regardless of the initial speed provided of course that the rate of acceleration remains constant.

Neither of equations 11 or 12 contain any term concerning the speed of the vehicles. This shows that the time of impact is completely independent of the speed of the vehicles. It does however depend of the acceleration, reaction time and initial gap.

The Highway Code in the UK contains tables of total stopping distances, consisting of reaction distance and emergency stopping distances. The perception – response time calculated using the Highway Code data is around 0.67 seconds which is considered by many researchers as particularly low. Olsen[2] for example suggests that 1.0 to 1.5 seconds is more realistic and may be even longer.

The Highway Code stopping distances assume a rate of deceleration of around 6.5 ms^{-2}, for a good dry road.

As discussed previously the UK Government ran a road safety campaign where the main thrust was to persuade drivers to adopt a two second gap between themselves and the vehicle they were following. The UK Highway Code now also incorporates the two second rule as general advice.

Using the Highway Code data and the react and stop solution discussed, it can be shown that the two second rule only applies at a speed of about 64 km/h (40 mph). At lower speeds a shorter time gap is required and at higher speeds a longer time gap is required. With the reaction only solution, a two second time gap adequately caters for the majority of drivers.

It remains open to question therefore whether the writers of the UK Highway Code, were considering following distances based on reaction or just picked a reasonable speed and derived the two second rule around that speed.

Acknowledgements

My thanks are due to Colin Dobbins of North Wales whose original question on this type of collision provided the inspiration for this paper.

REFERENCES

1 Neades, J. and **Ward, R.** Forensic Collision Investigation Manual. AiTS, 1996
2 Olsen, P L. Driver Perception Response Time. 1997 Proceedings of 3rd National ITAI Conference

Appendix A

Spreadsheet solution to worked example

Closing Speed Collision Calculations

Initial speed	20	m/s	**Impact?**	**Impact Time**			**Impact Speeds**			
Initial gap	4	m	Yes	1.082	s		**Veh 1**	12.565	m/s	
Reaction time	1	s	Quadratic				**Veh 2**	19.435	m/s	
Acceleration	-6.87	m/ss		**Distance**			**Difference**	6.870	m/s	
				17.622	m					
Linear	1.079112	s								
u * tr	20	m/s